L. Mac Intosh

No 13

SHORT SYSTEM OF

PRACTICAL ARITHMETIC,

COMPILED

FROM THE BEST AUTHORITIES ;

WITH

DEMONSTRATIONS OF THE RULES,

TO WHICH IS ANNEXED

A SHORT PLAN OF BOOK KEEPING,

THE WHOLE DESIGNED

FOR THE USE OF SCHOOLS.

By WILLIAM KINNE, *A. M.*

SECOND EDITION.

DISTRICT OF MAINE—TO WIT:

BE IT REMEMBERED, that on the tenth day of September, in the thirty second year of the Independence of the United States of America, EZEKIEL GOODALE of the said District, hath deposited in this Office the title of a Book, the right whereof he claims as Proprietor, in the words following, to wit ; " *A short system of Practical Arithmetic, compiled from the best Authorities; with Demonstrations of the Rules. To which is annexed a short plan of Book Keeping. The whole designed for the use of Schools.*"——In conformity to the act of the Congress of the United States, entitled " An Act for the encouragement of learning by securing the copies of Maps, Charts and Books to the Authors and Proprietors of such copies, during the times therein mentioned." And also " An Act supplementary to an Act, entitled An Act for the encouragement of learning by securing the copies of Maps, Charts and Books to the Authors and Proprietors of such copies, during the times therein mentioned, and extending the benefits thereof to the Arts of Designing, Engraving and Etching Historical and other Prints."

H. SEWALL, Clerk Dis. Court, Maine.
A true Copy as of Record,
Att. H. SEWALL, Clerk.

RECOMMENDATIONS.

Hallowell, December 3d, 1807.

HAVING at the request of EZEKIEL GOODALE, inspected the following " Short System of PRACTICAL ARITH-METIC," published by him, we are of opinion that it will be found an useful assistant in our Schools; and sincerely wish that the Author may be rewarded by meeting with due encouragement.

SAMUEL MOODY.
JOHN SEWALL.
E. GILLET.
SAM. S. WILDE.

Augusta, November 28th, 1807.

THE undersigned having examined " A Short System of PRACTICAL ARITHMETIC," published by EZEKIEL GOODALE of Hallowell, are of opinion that the work is executed with skill and judgment, and will prove a valuable assistant to the Instructors and Students of our Schools and Academies. And we sincerely wish the Author may be remunerated for his labour, by a rapid sale of the work.

DANIEL CONY.
DANIEL STONE.
WM. BROOKS.
HENRY SEWALL.

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TO THE FIRST EDITION.

. . . .

IF the state of an Art or Science were
to·be estimated from the multiplicity of books respect-
ing it, it'might be presumed, that Arithmetic had already
arrived at perfection. But a conclusion far different
from this will be made from a slight attention to the
publications on this subject, as they will be found but lit-
tle more than bare copies of each other, badly arranged,
and encumbered with such a variety of ill digested and
miscellaneous remarks, as embarrass the learner, and ren-
der them unfit for the purpose of teaching.

PERSPICUITY and conciseness are peculiarly requisite
in all works intended for instruction ; but in none more
so than in that of Arithmetic.

WITH a view to remove the obstacles which oppose
the progress of the learner, and to lessen the labors of the
instructor, all remarks with the demonstrations of the rules,
in the following treatise, are thrown into the form of notes ;
and nothing has been permitted to enter the text but a
set of exercises, preceded by directions for their perform-
ance, suitable for the learner to transcribe and fix in his
memory.

THE great advantages to be derived from considering
Arithmetic as a Science as well as an Art, are too obvious
to need a recital ; and the propriety of the arrangement
will be best felt by those who are engaged in the fatiguing
buisness of instructing.

THIS compilation, the design of which is to furnish
our public Schools with a methodical, plain and compre-
hensive system of practical Arithmetic, has been made
principally from the works of HUTTON, LUDLAM and BE-
ZOUT, with such alterations as the case seemed to require.

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TO THE *SECOND* EDITION.

. . . . . .

THE extensive circulation and rapid sale of this work, the whole of the first edition having been disposed of the last year, are sufficient testimonials of the high estimation in which it is held by the public. Such a plain, concise, comprehensive, and at the same time cheap system of Arithmetic was very much needed in schools ; and the frequent calls for it, since the edition sold off, shew that it has no adequate substitute.

THIS *second* edition has received some additions, emendations and corrections ; and is now, it is presumed, as free from errors as any production of the kind extant. The Publisher, confident of the merits of the work, and encouraged by former indulgence, issues a large edition, not doubting of its success with a discerning public.

                                EZEKIEL GOODALE.

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CONTENTS.

EXPLANATION of CHARACTERS.

= SIGNIFIES *equality* ; as 100 cents = 1 dollar, signi-
fies that 100 cents are equal to one dollar.

+ Signifies *plus*, or addition ; as 4 + 2 = 6.

— Signifies *minus*, or subtraction ; as 6 — 2 = 4.

× *Into* signifies multiplication ; as 3 × 2 = 6.

÷ By, or) (signifies division ; as 6 ÷ 2 = 3 or 2)6(3.
Division may also be denoted by placing the dividend
over a line, and the divisor under it ; thus, $\frac{6}{2}$ = 6 ÷ 2 = 3.

∴ ∷ ∴ Signifies arithmetical proportion ; thus, 2 ... 4
:: 6 .. 8.

∶ ∷ ∶ Signifies geometrical proportion ; thus, 2 : 4 : :
3 : 6 which is read, as 2 to 4, so is 3 to 6,

$\rceil^2$ Signifies the *second power* or square.

$\rceil^3$ Signifies the *third power* or cube.

$\rceil^n$ Signifies *any power*.

$\sqrt{}$, or $\rceil^{\frac{1}{2}}$ Signifies *the square root*.

$\sqrt[3]{}$, or $\rceil^{\frac{1}{3}}$ Signifies *the cube root*.

$\sqrt[n]{}$, or $\rceil^{\frac{1}{n}}$ Signifies *any root*.

$\rceil^{\frac{m}{n}}$ Signifies *any root* of *any power*.

ARITHMETIC.

A RITHMETIC is the art and science * of numbers, and has for its operations four fundamental rules, viz. *Addition, Subtraction, Multiplication* and *Division.* To understand these, it is necessary to have a perfect knowledge of our method of Numeration or Notation.

NOTATION

TEACHES to express numbers by words or characters.

When performed by means of characters or figures, ten are employed. Nine of these are of intrinsic value and are called digits, being written and named thus :

1 one.	4 four.	7 seven.
2 two.	5 five.	8 eight.
3 three.	6 six.	9 nine.

The tenth figure, namely 0, is called *zero* or cypher, and denotes a want of value wherever it is found.

Besides the simple value of the digits as noted above, they have each a local one, which depends on the following principle.

In a combination of figures, reckoning from right to left, the figure in the first place represents its simple value ; that in the second place ten times its simple value ; that in the third place an hundred times its simple value ; and so on ; each figure acquiring anew a ten fold value for every high- er place it occupies. Hence our system of arithmetic is called *decimal.*

The names of places are denominated according to their order. The first is the place of units ; the second of tens ; the third of hundreds ; the fourth of thousands ; the fifth of ten thousands ; the sixth of hundred thousands ; the seventh of millions ; and so on. Thus in the number 8888888 ; 8 in the first place signifies only eight ; 8 in the

---

* It is a science, as it explains the nature and quality of numbers, and demonstrates the reason of practical opera- tions ; and an art, as it shows the method of operating.

second place eight tens or eighty ; 8 in the third place
eight hundred ; 8 in the fourth place eight thousand ; 8 in
the fifth place eighty thousand ; 8 in the sixth place eight
hundred thousand ; 8 in the seventh place eight millions.
The whole number is read thus, eight millions, eight hundred
and eighty eight thousand, eight hundred and eighty eight.

Though a cypher has no value of itself, yet it occupies
a place ; and when set on the right hand of other figures
it increases their value in the same tenfold proportion :
Thus in the number 8080 ; the cyphers in the first and third
places denote, that, though no simple unit or hundreds are
reckoned, yet the place of units and that of hundreds are to
be kept up to assist in reckoning the tens and thousands.
The above number (8080) is read eight thousand and eighty,
which without the two cyphers would be read eighty eight.

Large numbers are divided into periods and half peri-
ods, each half period consisting of three figures. The
name of the first period is units ; of the second millions ;
of the third billions ; of the fourth trillions ; and also, the
first part of any period is so many units of it ; and the lat-
ter part so many thousands of it.

EXAMPLE.

Periods.	Trillions.		Billions.		Millions.		Units.	
	4.		3.		2.		1.	
Half do.	thou.	units.	thou.	units.	thou.	units.	thou.	c. x. u.
Figures.	137	462	572	329	484	617	291	387

Read as follows :

One hundred and thirty seven thousand, Four hundred and sixty two trillions ; Five hundred and seventy two thousand, Three hundred and twenty nine billions ; Four hundred and eighty four thousand, Six hundred and seventeen millions ; Two hundred and ninety one thousand, Three hundred and eighty seven.

APPLICATION.

To express in Figures, Numbers which exceed nine.

RULE. Write the digits denoting each part of the number in their proper places ; and where a digit is wanting; put a cypher to mark the deficiency.

EXAMPLES.

Twenty five, ... 25
One hundred, ... 100
Three thousand and fifteen, ... ˙3015
Eight hundred and twelve thousand, . . . 812000
Thirty one thousand, two hundred and six, . 31206
Six millions, seven thousand and eight, . . 6007008

To read NUMBERS.

RULE. To the simple value of each figure, join the name of its place ; beginning at the first place to the left.

EXAMPLES.

64, Sixty four.
396, Three hundred and ninety six.
4015, Four thousand and fifteen.
76920, Seventy six thousand nine hundred and twenty.
104080, One hundred and four thousand and eighty.
5300648, Five millions, three hundred thousand, six hundred and forty eight.

ADDITION.

ADDITION in Arithmetic is the uniting or joining together of two or more numbers.

SIMPLE ADDITION is the collecting of several numbers of the same denomination into one sum.

SIMPLE ADDITION.

RULE.* Write the numbers, units under units, tens under tens, &c. and draw a line under the whole. Add up the unit column, and if the sum be less than ten, write it under the column ; if it be ten or any number of tens, write a cypher ; if there be an excess over ten or tens write down this excess, and carry as many units to the next column, as there are tens ; † and thus pro-

---

* The rule and proof depend on the axiom, that " the whole is equal to the sum of all its parts."

† This is caused by the value of figures increasing from the *right to the left* hand in a tenfold proportion, as may be *seen under notation.*

ceed with each remaining column, writing the whole sum
under the last.

Proof. Draw a line below the upper number, add the
remaining numbers, as shown in the rule ; add the sum
thus found and the upper number together, and if the sum
be equal to the first addition, the work is right.

EXAMPLES.

1.	2.	3.
2178	567842	321674
4216	143469	92167
3945	782107	8547
2763	695213	26
1684	203169	2141
14786		
12608		
14786		

PROOF *by casting out the* NINES.

RULE.* Add the figures in the upper number together ;
divide this sum by nine, and set the remainder opposite
the number to the right hand ; do the same with each of
the numbers ; then if the excess of nines in the sum of the
remainders be equal to the excess of nines in the sum total,
the work is supposed to be right.

EXAMPLE.

94346	Excess of nines			8
42130	.	.	.	1
61679	.	.	.	2
198155	.	.	.	2

* REASON OF THE RULE. As a figure, by its removal from
one place to the next higher takes to itself nine times its val-
ue, it follows, that any figure, however far removed from the
place of units and divided by 9, will leave only a remainder
of itself ; therefore any number and the sum of the digits of
that number, divided by 9, will leave equal remainders.

This additional value, taken by figures, by their removal
to a higher place, gives to 9 the power of producing an ef-
fect, which cannot belong to any other of the digits except
3, and to this only as it is an even part of 9.

SIMPLE SUBTRACTION.

SUBTRACTION is finding the difference of two numbers, by taking the less from the greater. It is simple subtraction if the numbers are of one denomination.

The greater number is called the *minuend ;* the less the *subtrahend ;* and the number found by the operation, the difference or *remainder.*

RULE.* Write the less number under the greater, placing units under units, tens under tens, &c. and draw a line under them. Take each figure in the subtrahend from its corresponding one in the minuend, and set down the remainder. If the lower figure be greater than the one above it, add ten to the upper figure, from which sum take the lower, and set down the remainder, carrying one to the next lower figure; and thus proceed until the whole is finished.

PROOF. Add the *remainder* to the *subtrahend,* and if the sum be equal to the *minuend,* the work is right.

EXAMPLES.

	1.		2.
From	67216	the minuend,	46132941
Take	43792	the subtrahend.	17316257
	23424	Remainder.	
	67216	Proof.	

	3.	4.	5.
	71290	30421	87652176
	46172	10604	9107215

* DEMONSTRATION. When all the figures of the lower number are less than their corresponding ones in the upper, the difference of the several figures, as they stand, in their corresponding places, must altogether make the true difference ; for as the sum of the parts is equal to the whole, so must the sum of the differences of all the similar parts be equal to the difference of the whole. The adding of ten to the minuend and carrying one to the next figure in the subtrahend is, by the nature of numeration, only adding equal

B

SIMPLE MULTIPLICATION.

MULTIPLICATION is finding the amount of any given number, by repeating it any proposed number of times.

The number to be multiplied is called the *multiplicand*.

The number which multiplies is called the *multiplier*.

The number arising from the operation is called the *product*.

The multiplicand and multiplier are called *factors* ; and if these are of one denomination it is called Simple Multiplication.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

USE *of the* TABLE *in* MULTIPLICATION.

Find the multiplier in the left hand column, and the multiplicand in the uppermost line ; and the product is in the common angle of meeting, or against the multiplier, and under the multiplicand.

---

quantities to each ; and is done to resolve the minuend into such parts, as are each equal or greater than the corresponding parts of the subtrahend.

The method of proof is evidently correct ; for the difference of two numbers, added to the less, makes the less equal to the greater. Q. E. D.

RULE.* Write the multiplier under the multiplicand, so that units may stand under units, tens under tens, &c. and

* DEMONSTRATION. It is plain that the product, arising by multiplying every part of the multiplicand by the unit figure of the multiplier, will be a number containing the multiplicand so many times repeated as that figure contains units, or the true product for the first figure of the multiplier; for by multiplying every part the whole is multiplied. The reason of setting down both the product when less than ten, and the excesses of tens, under the figure multiplied, and carrying forward an unit for every ten found, to the product of the next place, is the same as that given under addition.

As the second figure of the multiplier occupies the place of tens, the product must be ten times its simple value, and therefore the first figure of this product must be put in the place of tens, or directly under the multiplying figure; and so of the rest. Hence the sum of these several products will be equal to the required product; for the whole of the multiplicand is thus multiplied by the whole of the multiplier. Q. E. D.

The reason of the method of proof depends upon the proposition, that if two numbers are to be multiplied together, either of them may be made the multiplier, and the product will be the same.

PROOF *by casting out the* NINES.

RULE. Find the excess of nines in the factors; multiply the excesses together, and if the excess of nines in their product be equal to the excess of nines in the total product, the work is supposed to be right.

EXAMPLE.

```
63142   7=excess of nines in the multiplicand.
  654   6=     ditto        in the multiplier.
 ─────
 252568
 315710
 378852
 ─────
41294868   6=     ditto        in the product=to
                 excess of nines in 6×7.
```

draw a line under them. Begin at the right hand and multiply the whole multiplicand severally by each figure in the multiplier, setting down the first figure of every line directly under the figure used as a multiplier, and carrying for the tens as in addition. Add all the lines together, and their sum is the product.

PROOF. Make the multiplicand the multiplier, and proceed as before; and if this product be equal to the former the work is right.

EXAMPLES.

1.

```
  4271 Multiplicand.          329 excess of nines 5
   329 Multiplier.           4271  ..  ..  ..    5
```
———————————————————————

```
 38439                        329
  8542                       2303
12813                         658
—————                        1316
1405159 Product.            —————
```
Proof 1405159 excess of nines 7 Proof.

```
       2.                            3.
   5124167                        42179416
        3                                5
```
—————————   —————————

Pro. 15372501

—————————————————————————————————

DEMONSTRATION OF THE RULE. Let each of the factors be resolved into two parts; one containing the whole number of nines, and the other their remainders or their excesses of nines. The product of the first part will contain an exact number of nines, and therefore can in no way affect the excess of nines, if the product of the second part be considered with it. The product of the second part will be the same, whether it be separately considered, or incorporated with the first part; and consequently the excess of nines in each product will be the same: But in one product it represents the excess of nines in the product of the excesses of nines in the factors, and in the other the excess of nines in the total product. Q. E. D.

4.	Multiply 74216	by	2	product 148432
5.	— 49627		6	297762
6.	7689657		7	53827599
7.	3912461		8	31299688
8.	2674876		9	24073884
9.	427691		10	4276910
10.	716284		11	7879124
11.	567295		12	6807540
12.	691861		26	17988386
13.	129186		98	12660228
14.	281216		978	275029248
15.	181281		763	138317403
16.	281691		76286	21489079626
17.	264648436		3639604	963215506259344

CONTRACTIONS IN MULTIPLICATION.

CASE I. *When there are cyphers at the right hand of one or both of the factors.*

RULE. Proceed as before, neglecting the cyphers, and to the right hand of the product, place as many cyphers as are in both the numbers.

EXAMPLES.

1.	2.	3.
27600	180120	27640
48000	48100	20
2208		
1104		
1324800000	8663772000	

CASE II. *When the multiplier is the product of two or more numbers.*

RULE. Multiply once successively by each of those numbers instead of using the whole multiplier at once.

EXAMPLES.

1.
Multiply 7629 by 63
7×9=63   7

53403
9

Prod. 480627

2.
74639 by 72

3.
46217 by 96

4.
37692 by 132

B 2

EXAMPLES.

	1.			2.	
Divis.	Divid.	Quot.	Divis.	Divid.	Quot.

```
  48)7641312(159194          5)172164(34432 4/5
     48            48           15
     ──           ──           ──
     284        1273552         22
     240         636776         20
     ──         ───────         ──
     441       7641312 Proof.   21
     432       ────────         20
     ──                         ──
      93                        16
      48                        15
      ──                        ──
     451                        14
     432                        10
     ──                         ──
     192                        4 Remainder.
     192                        ─
     ──
```

```
          3.
       6)7241324(            Or 5)172164(
```

Quo. 1206887 2/3 Rem.   Quo. 34432 4/5 Rem. 4

NOTE. If there be no remainder the quotient is the perfect answer to the question ; but if there is, to complete the quotient, put the remainder at the end of it and the divisor below it, drawing a line between the two.

				Quotient.	Rem.
4.	Divide 3764592	by	7	Ans. 537798	and 6
5.	527684		8	65960	4
6.	1410217		9	156690	7
7.	612948		11	55722	6
8.	317926		12	26493	10
9.	153598		29	5296	14
10.	301147		63	4780	7
11.	138317403		763	181281	0
12.	11214887		232	48340	7
13.	4678216		400	11695	216
14.	1030603615		3215	320561	0
15.	4917968967		2359	2084768	1255
16.	210634711		6000	35105	4711

CONTRACTIONS.

CASE I. When there are cyphers at the right hand of
the divisor, cut them off; likewise cut off the same num-
ber of digits from the right hand of the dividend; then di-
vide as usual, and to the remainder annex the digits cut off
from the dividend.

EXAMPLES.

<pre>
        1.   2.
 342,00)6792,16(19   135,000)27619,413(204
        342   270
        ————   ————
        3372   619
        3078   540
        ————————   ————
          29416 Remainder.   79413 Rem.
</pre>

CASE II. If the divisor be a product of two or more
numbers, divide continually by those numbers instead of
the whole at once.

EXAMPLES.

<pre>
         1.   2.
  Divide 7621460 by 16   4792161 by 48
     4)7621460   6)4792161
     ————————   ————————
     4)1905365   8)798693—3
     ————————   ————————
   Quo. 476341=4 Rem.   99836—5×6+33
</pre>

NOTE. It sometimes happens that there is a remainder
to each of the quotients, and neither of them the true one;
but the true remainder may be found by the following rule.

RULE. Multiply the last remainder by the last divisor
but one, and to the product add the preceding remainder;
multiply this sum by the next preceding divisor, and to this
product add the next preceding remainder, and so on, until
all the remainders and divisors are used; and the last sum
will be the true remainder.

<pre>
            1.   2.
      Divide 6421671 by 448   Divide 27162 by 63
8×8×7=448 8)6421671
           ————————
           8)802708—7
           ————————
           7)100338—4
           ————————
      Quotient 14334—4×8+7=39 Remainder.
</pre>

*PRACTICAL QUESTIONS under the PRECEDING
RULES.*

1. Add fourteen thousand, five hundred and nine ; one thousand, nine hundred and twenty one ; six hundred and twenty thousand, three hundred and forty-seven ; and five million, twenty-three thousand, and nineteen, together.
                                         Ans. 5659796 sum.

2. What is the sum of 76129+54216+39127+62357+514026 ?   Ans. 745855.

3. What is the difference between four million two hundred and ten thousand and twelve ; and six hundred and fifty-nine thousand seven hundred and ninety-seven ?
                                         Ans. 3550215.

4. Take nine hundred and one thousand and fifteen, from one million, one thousand, one hundred and one.
                                         Ans. 100086.

5. A farm of 460 acres is let for 2 dollars per acre ; how much does the rent amount to ?   Ans. 920 dollars.

6. If a man's income be 6 dollars a day, how much does it amount to in a year, allowing 365 days in a year?
                                         Ans. 2190 dollars.

7. What is the product of 376×54 ?   Ans. 20304.

8. 64 men have 17280 dollars divided equally among them ; what is each man's part ?   Ans. 270 dollars.

9. Multiply three hundred and seventy-eight thousand, and five hundred, by thirty-four. Ans. 12869000 product.

10. What is the third part of 3669 ?   Ans. 1223.

11. Divide 6764 by 19.   Ans. 356 quotient.

12. What number must be added to 764 to make it 1256 ?   Ans. 492.

13. By what number must I multiply 67, that the product may be 871 ?   Ans. 13.

14. There are two numbers whose difference is 796, the greater number is 4320 ; I demand the less.   Ans. 3524.

15. Supposing a man to have been born in the year 1762 ; how old was he in 1806 ?   Ans. 44.

16. Suppose a man to have been 78 years old in the year 1806 ; in what year was he born?   Ans. 1728.

17. What will 12 tons of hay come to at 27 dollars per ton ?   Ans. 324 dollars.

18. What will 750 barrels of beef come to at 11 dollars per barrel ; and what will the profits amount to in selling it, if I clear 3 dollars on each barrel ?

Ans. $\begin{cases} 8250 \text{ dollars amount.} \\ 2250 \text{ dollars do. profit.} \end{cases}$

19. There is a town which contains 290 houses, and each house 6 inhabitants ; how many inhabitants are there in that town ?   Ans. 1740.

20. A prize of 48726 dollars is owned by 270 men ; what is each man's share ?   Ans. 180$\frac{21}{45}$ dollars.

21. If 12 bundles of wheat produce 1 bushel, how many bushels will 4764 bundles produce ?   Ans. 397 bushels.

22. If 36 be added to 97 and subtracted from 250, the remainder multiplied by 12 and the product divided by 6 ; what will the quotient be ?   Ans. 234.

. . . . . .

TABLES of MONEY, WEIGHTS and MEASURES.

1. MONEY.

4 Farthings make one penny   *qr.*   *d.*   denote farthings and pence respectively.
12 Pence make one shilling . .   *s.*   . . .   *Shilling.*
20 Shillings : . . . . *£.* . . . .*Pound.*

$\frac{1}{4}$ Is one farthing or one fourth ; $\frac{1}{2}$ is one halfpenny, or one half ; $\frac{3}{4}$ three farthings, or three fourths.

2. TROY WEIGHT.

24 Grains make one pennyweight, marked *grs. dwt.*
20 Pennyweights . . 1 Ounce, ... *oz,*
12 Ounces . . . . 1 Pound, ... ℔.

By this weight are weighed jewels, gold, silver, electuaries and liquors.

3. APOTHECARIES WEIGHT.

20 Grains make . . . . 1 Scruple, marked . . *gr.* ℈.
3 Scruples . . . . . . 1 Dram, ... ℥.
8 Drams . . . . 1 Ounce, ... ℥.
12 Ounces . . . . . . 1 Pound, ... ℔.

Apothecaries use this weight in compounding their medicines, but they buy and sell their drugs by Avoirdupois weight.

4. AVOIRDUPOIS WEIGHT.

16 Drams make	. . .	1 Ounce, marked	. *dr. oz.*
16 Ounces	. . . . .	1 Pound,	. . . . . ℔.
28 Pounds	. . . . .	1 Quarter,	. . . . *qr.*
4 Quarters	. . . .	1 Hundred weight,	. . *cwt.*
20 Hundred wt.	. . .	1 Ton,	. . . . . . *T.*

By this weight are weighed all things of a coarse nature; such as leather, cheese, grocery wares, and all metals except gold and silver.*

– NOTE. 5760 grains=1 ℔ Troy ; 7000 grains=1 ℔ Avoirdupois ; therefore the weight of a pound Troy is to a pound Avoirdupois as 5760 to 7000, or as 144 to 175.

5. CLOTH MEASURE.

4 Nails make	. . .	1 Quarter, marked	*na. qr.*
4 Quarters	. . . .	1 Yard,	. . . . . . *yd.*
3 Quarters	. . . .	1 Ell Flemish,	. . *E. Fl.*
5 Quarters	. . . .	1 Ell English,	. . *E. E.*
6 Quarters	. . . .	1 Ell French,	. . *E. Fr.*

6. LONG MEASURE.

3 Barley Corns make	. .	1 Inch, marked	. *Bar. In.*
12 Inches	. . . . .	1 Foot,	. . . . . *Ft.*
3 Feet	. . . . .	1 Yard,	. . . . . *Yd.*
5½ Yards, or 16½ Feet	. .	1 Pole, Rod or Perch,	*Rod.*
40 Poles or Rods	. . .	1 Furlong,	. . . . *Fur.*
8 Furlongs	. . . . .	1 Mile,	. . . . *Mi.*
3 Miles	. . . . .	1 League,	. . . *Lea.*
60 Geographical, or } 69½ Staute Miles }	. .	1 Degree,	. . . *Deg.*

By this measure distances are measured.

* A Firkin of Butter is 56℔.	A Quintal of Fish 112℔.
A Firkin of Soap . . . 64.	A Stone of Iron . . . 14.
A Barrel of Beef . . . 220.	A Gallon of Train Oil 7½.
. . . Pork . . . 220.	20 Things make 1 Score.
. . . Potashes . . 400.	12 do. . . . 1 Dozen.
. . . Soap . . . 256.	12 Dozen . . 1 Gross.
. . . Butter . . 224.	144 do. . . 1 Great Gross.

7. LAND OR SQUARE MEASURE.

144 Square Inches make 1 Square Foot.
9 Feet . . . . . . 1 Yard.
30¼ Yards or } . . . 1 Rod, Pole or Perch.
272¼ Feet }
40 Rods . . . . . 1 Rood or quarter of an Acre.
4 Roods . . . . 1 Acre.

By this measure surfaces are measured.

8. CUBIC OR SOLID MEASURE.

1728 Solid Inches make . . . 1 Foot.
27 Feet . . . . 1 Yard.
40 Feet of round, or } . . 1 Ton or Load.
50 Feet of hewn Timber }
128 Feet, i. e. 8 feet in length } =1 Cord of Wood.
4 in breadth and 4 in height }

By this measure the contents of solids are obtained.

9. DRY MEASURE.

2 Pints make . . . 1 Quart.
4 Quarts . . . . 1 Gallon.
2 Gallons . . . . 1 Peck.
4 Pecks . . . . . 1 Bushel.
8 Bushels . . . . 1 Quarter.
36 Bushels . . . . 1 Chaldron.

NOTE. The diameter of the Winchester or common bushel is 18½ inches, and its depth 8 inches.

The gallon dry measure contains 268⅘ cubic inches.

10. WINE MEASURE.

4 Gills make . . 1 Pint, marked . . *Gill, Pt.*
2 Pints . . . . 1 Quart, ... *Qt.*
4 Quarts . . . . 1 Gallon, ... *Gal.*
42 Gallons . . . . 1 Tierce, ... *Tier.*
63 Gallons . . . 1 Hogshead, ... *Hhd.*
84 Gallons . . . 1 Puncheon, ... *Punch.*
2 Hogsheads . . 1 Pipe or Butt, . . . *Pipe.*
2 Pipes or 4 Hhds. 1 Tun, ... *Tun.*

NOTE. The wine gallon contains 231 cubic inches.

C

11. Ale Measure.

2 Pints make	. .	1 Quart, marked	. .	*Pt. Qt.*
4 Quarts	. . .	1 Gallon,	. . . . . ,	*Gal.*
8 Gallons	. . .	1 Firkin of Ale,	. . .	*A. Fir.*
9 Gallons	: . .	1 Firkin of Beer,	. .	*B. Fir.*
36 Gallons	. . .	1 Barrel,	. . . . .	*Bar.*
54 Gallons	. . ..	1 Hogshead,	. . . .	*Hhd.*
3 Barrels	. . .	1 Butt,	. . . .	*Butt.*

NOTE. The ale gallon contains 282 cubic inches.

12. Time.

60 Seconds make	. . .	1 Minute, marked	*S. M.*
60 Minutes	. . . . .	1 Hour,	. . . . . *H.*
24 Hours	. . . . . .	1 Day,	. . . . . *D.*
7 Days	. . . . . .	1 Week,	. . . . *W.*
4 Weeks	. . . . .	1 Month,	. . . . *Mo.*
13 Lunar or 12 Solar Months, or } 365 Days . . . . }		1 Year,	. . . *Y.*

NOTE. 365 days 5 hours 48 minutes 57 seconds, make a solar year, according to the most exact observation.

April, June, September and November, have each 30 days ; each of the other months has 31, except February, which has 28 in common years and 29 in leap years.

13. Circular Motion.

60 Seconds make	. . .	1 Prime minute, marked	*'' '*
60 Minutes	. . . .	1 Degree,	. . . . . Θ
30 Degrees	. . .	1 Sign,	. . . . . . S
12 Signs, or } 360 Degrees }	. . .	{ The whole circle { of the Zodiac.	

. . . .

REDUCTION.

REDUCTION teaches to change the denomination of numbers without altering their value.

RULE.* When the reduction is from a higher denomination to a lower, multiply the highest denomination by as

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* The reason of the rule is plain. The highest denomination is considered as an integer, made up of a certain number of parts of the next lower ; therefore multiplying the highest denomination by such a number as composes

many of the next lower as make one of the highest, adding
to the product the parts of the same name ; multiply this
sum by the next lower, adding to the product the parts of
its own name if any ; and so on to the denomination required.

When the reduction is from a lower to a higher denomi-
nation, divide the given number by as many of that denom-
ination as make one of the next higher, and so on, to the de-
nomination required ; and the last quotient with the seve-
ral remainders (if any) will be the answer required.

The proof is had by reversing the question.

EXAMPLES.

MONEY.

1. In 476 pounds, how many shillings and pence ?

$$476$$
$$20$$

$$9520 \text{ Shillings.}$$

Ans. 114240 Pence.

12   12)114240

20)952,0

Proof. 476

2. In 3694 shillings, how many pence ? Ans. 44328.
3. How many farthings in 69217 pence ?

Ans. 276868.

4. Reduce 6942 pounds to farthings.

Ans. 6664320.

5. In £.49 19s. 11¾d. how many farthings ?

£. s. d. qr.
49 19 11 3
20

999 Shillings.
12

11999 Pence.
4

Ans. 47999 Farthings.

it, can only change its name, but not its value ; and this
reasoning may be carried through all the denominations.
The contrary will readily be seen to hold true by division.

6. How many Pence in £.472 13s. 4d. ?

Ans. 113440.

7. How many pounds in 467216 farthings ?

4)467216

————

12)116804

————

20)9733 8d.

————

Ans. £.486 13s. 8d.

8. How many pounds in 9752 pence ? Ans. £.40 12s. 8a.
9. In 648 guineas, * how many pence ? Ans. 217728.
10. How many guineas in 28560 pence ? - Ans. 85.
11. In 37 pistoles, how many farthings ?

Ans. 39627.

12. In 48960 farthings, how many pence, three-pences, six-pences and dollars ?

Ans. 12240 pence, 4080 three-pences, 2040 six-pences and 170 dollars.

13. In 427 moidores, how many dollars and pounds ?

Ans. $2562 £.768 12s.

14. In 11040 pence, how many dollars ? Ans. $153 2s.
15. How many pounds in 91751 farthings ?

Ans. £.95 11s. 5¾d.

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* TABLE.

The weight and value of several pieces of English, Portuguese and French Coins.

		£.	s.	d.		$.	Cts.
An English Shilling is			1	4	Or		22⅔
A French Franc, ...			1	1½	.		18¾
A Livre Tournois, ...			1	1½	.		18¾
An English or French Crown, . .			6	7⅓	.	1	10
Napoleon, 4pwt.6grs. . . .	1	2	6	.	.	3	75
½ Johannes, . 9	2	8	0	.	.	8	00
Moidore, . 6 18	1	16	0	.	.	6	00
Eng. Guinea, 5 6	1	8	0	.	.	4	66⅔
French do. 5 5	1	7	6	.	.	4	58⅓
Pistole, . . 4 5	1	2	3¾	.	.	3	72

The gold coins of Great Britain and Portugal are estimated at 27 grains to a dollar; those of France and Spain at 27⅘ grains to a dollar.

TROY WEIGHT.

1. How many grains in 47℔ 10 oz. of gold?

Ans. 275520.

2. In 47128 grains of gold, how many pounds?

Ans. 8℔. 2 oz. 3pwt. 16grs.

3. In 5605 grains, how many ounces?

Ans. 11oz. 13pwt. 13grs.

4. How many grains in 18℔. 5oz. 9pwt. 21grs. ?

Ans. 106317.

AVOIRDUPOIS WEIGHT.

1. In 12 cwt. 1 qr. 18 ℔. how many ounces?

Ans. 22240.

2. How many tons in 3440640 drams? Ans. 6 Tons.

3. In 2 T. 15 cwt. 2 qrs. 17℔. how many pounds?

Ans. 6233℔.

4. How many pieces of 4 ℔. 5½ ℔. and 6½ ℔. of each an equal number, in 62 cwt. 2 qrs. 24 ℔. of beef?

Ans. 439 pieces of each.

APOTHECARIES WEIGHT.

1. In 31 ℔. 2 ℥. 6 ʒ. how many drams?

Ans. 2998.

2. How many pounds in 2535 scruples?

Ans. 8 ℔. 9 ℥. 5 ʒ.

CLOTH MEASURE.

1. How many quarters in 83 yds. 3 qrs.

Ans. 335.

2. In 2528 nails, how many yards? Ans. 158.

3. In 840 nails, how many ells english? Ans. 42.

4. How many yards of Holland in 58 pieces, each containing 36 ells flemish? Ans. 1566.

LONG MEASURE.

1. In 70 miles, how many furlongs and poles?

Ans. 560 fur. 22400 poles.

2. How many leagues in 21120 yards? Ans. 4.

3. How many barley corns in 360 degrees, each degree 69½ miles? Ans. 4755801600.

4. How often will a wheel that is 15 feet in circumference, turn round in the distance from Hallowell to Farmington, it being 32 miles? Ans. 11264.

C 2

LAND OR SQUARE MEASURE.

1. In 17 acres 3 roods 10 poles, how many poles ?
<div align="right">Ans. 2850.</div>

2. In 815443200 inches, how many acres ?
<div align="right">Ans. 130.</div>

3. How many acres in 6654 rods or poles?
<div align="right">Ans. 41 *Acres,* 2 *Roods,* 14 *Rods.*</div>

4. In 6 acres 1 rood, how many perches ?
<div align="right">Ans. 1000.</div>

CUBIC OR SOLID MEASURE.

1. In 9 tons of round timber, how many inches ?
<div align="right">Ans. 622080.</div>

2. How many cords of wood in 3096576 inches ?
<div align="right">Ans. 14.</div>

3. In 259200 inches of hewn timber, how many tons?
<div align="right">Ans. 3.</div>

4. How many bricks 8 inches long, 4 wide and 2 thick, will build a house 44 feet long, 40 feet wide and 20 feet high, with walls 12 inches thick ?   Ans. 88560.

DRY MEASURE.

1. In 49 bushels, how many quarts ?   Ans. 1568.
2. How many bushels in 27072 quarts ?   Ans. 846.
3. How many pints in 150 bushels of corn ? Ans. 9600.

WINE MEASURE.

1. In 3 hogsheads how many gills ?   Ans. 6048.
2. How many hogsheads in 6480 gills ?
<div align="right">Ans. 3 *Hhds.* 13 *Gals.* 2 *Qts.*</div>

3. How many pints in 25 tuns of wine ?   Ans. 50400.
4. In 30876 gills, how many hogsheads ?
<div align="right">Ans. 15 *Hhds.* 19 *Gals.* 3 *Qts.* 1 *Pt.*</div>

BEER MEASURE.

1. In 10 hogsheads 17 gallons, how many gills ?
<div align="right">Ans. 17824.</div>

2. How many firkins of ale in 7624892 pints ?
<div align="right">Ans. 119138 *A. Fir.* 7 *Gals.* 2 *Qts.*</div>

3. How many pints in 12 hogsheads, 15 gallons, 2 quarts ?
<div align="right">Ans. 5308.</div>

4. In 6420 quarts, how many firkins of beer ?
<div align="right">Ans. 178 *B. Fir.* 3 *Gals.*</div>

TIME.

1. How many minutes in 347 days ?   Ans. 499680.
2. In 57953 hours, how many weeks ?
                               Ans. 344 *W.* 6 *D.* 17 *H.*
3. How many seconds are there in 72 years, 10 days, 18 hours, 11 minutes, allowing 365 days and 6 hours to a year ?   ,   Ans. 2273076660.
4. How many days from the 20th of April to the 16th of December following ?   Ans. 240.

CIRCULAR MOTION.

1. In 6 signs of the zodiac, how many seconds ?
                               Ans. 648000.
2. How many prime minutes in 360 degrees ?
                               Ans. 21600

. . . .

FEDERAL MONEY.*

THE denominations of Federal Money, like figures in whole numbers, increase in a tenfold proportion, beginning with mills, of which

       10 make   . .   1 Cent, marked *m. c.* respectively.
       10 Cents . . 1 Dime,   . . . *d.*
       10 Dimes . . 1 Dollar, . . *Doll. or.$.*
       10 Dollars .   1 Eagle, . . . *E.* '

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* Federal Money ought, in strict propriety, to be treated of after decimal fractions ; but usefulness (as fractions are not always understood) requires, and its simplicity and near alliance to whole numbers, will admit it in this place.

The coins of the United States are three of gold, six of silver and two of copper. The gold coins are called an eagle, half eagle and quarter eagle ; the silver, a dollar, half dollar, quarter dollar, double dime, dime and half dime ; and the copper, a cent and half cent.

The weight of the eagle is 11 pennyweights and 6 grains ; the weight of the dollar 17 pennyweights 8 grains ; of the dime, 1 pennyweight 17¾ grains ; of the cent 8 pennyweights 16 grains. The standard for gold coin is eleven parts fine gold and one part alloy ; the alloy consisting of silver and copper. The standard for silver is 1485 to 179

	$. c. m.				$. c. m.
3. Multiply	67,48 2	by	5	Product	337,41 0
4.	76,43		4		305,72
5.	3,16 4		9		28,47 6
6.	,78 1		12		9,37 2
7.	1,06		45		47,70
8.	3,16		150		474,00
9.	4,25		598		2541,50
10.	4,96 3		347		1722,16 1
11.	10,50		14½		152,25
12.	1,20		84¼		101,10
13.	2,40		26¾		64,20

DIVISION OF FEDERAL MONEY.

RULE. Write the number and divide as in simple divis-
ion. The quotient will be of the same denomination as the
lowest of the dividend.

EXAMPLES.

1.	2.	3.
$. c. m.	$. c.	$. c. m.
6)47,26 2	8)6914,24	11)7,49 1
7,87 7	864,28	68 1

4.

$. c. m.

237)6742,27 1(28448 mills, or 28,44 8

    474

    2002   5.

    1896 $. c. m. $. c. m.

            387)753,35 7(1,94 6

    1062

     948   6.

          359)259,23 7(,72 2

    1147

     948   7.

         475)74,10 0(,15 6

    1991

    1896

     95 Rem.

REDUCTION OF FEDERAL MONEY.

To reduce Dollars to Cents and Mills.

Multiply the dollars by 100 for cents, and the cents by
10 for mills; or to the dollars annex two cyphers for cents
and three for mills.

To reduce Mills and Cents to Dollars.

Divide the mills by 10 and the quotient will be cents;
divide the cents by 100 and the quotient will be dollars;
or if the number be cents, point off two, and if mills, three
figures on the right hand; then the figures on the left
hand of the comma will be dollars, the two first on the
right hand will be cents, and the third, if any, will be mills.

EXAMPLES.

1. Reduce 674 dollars to cents and mills.

$$674$$
$$100$$

$$\overline{67400} \text{ Cents.}$$
$$10$$

$$\overline{674000} \text{ Mills.}$$

Or 67400 Cents.
674000 Mills.

2. How many dollars in 642179 mills?

$$10)642179$$

$$\overline{100)64217-9}$$

$$\overline{\$ 642-17-9}$$
Or $ 642 17 c. 9 m. Ans.

3. How many mills in 47692 dollars? Ans. 47692000.
4. In 46791 cents, how many dollars? Ans.$467 91cts.
5. In 6421796 mills, how many dollars, cents and mills?
                                Ans. $ 6421 79 cts. 6m.

   To reduce New England currency to Federal Money.

   CASE I. If the sum consist of pounds only, annex three
cyphers to it and divide by 3; the quotient will be the an-
swer in cents.*

---

   * As a dollar is $\frac{6}{20}$ or $\frac{3}{10}$ of a pound, it is plain that an-
nexing a cypher to the pounds and dividing by 3, will give a

EXAMPLES.

1. Reduce £.3762 to Federal Money.

   3)3762000
   ————————
   1254000 Cents, *or*,
    $12540 Ans.

2. Reduce £.471 to Federal Money. Ans. $1570.
3. Reduce £. 37 to dollars and cents. Ans. $123 33⅓cts.
4. Reduce £.8 to dollars and cents. Ans. $26 66⅔cts.
5. Reduce £. 30 to dollars.   Ans. $100.
6. In £.47 how many dollars ?

   3)47000
   ————————
   156,66⅔
   ————————
  Ans. $156 66⅔cts.

7. How many cents in £.631 ? Ans. 210333⅓cts.
8. How many dollars in £.24 ?   Ans. $80.
9. How many dollars and cents in £.29 ?
    Ans. $96 66⅔cts. or 9666⅔cts.

CASE II. If pounds and shillings are given, to the pounds annex half the number of shillings and two cyphers, if the number of shillings be even ; but if the number be odd, annex half the even number, and then 5 for the odd shilling, and one cypher, and divide by 3 ; the quotient is the answer in cents.

EXAMPLES.

1. Reduce £.64 16s. to dollars and cents.

   3)64800
   ————————
  Ans. 21600 *cts. or* $216.

2. Reduce £.17 13s. to dollars and cents.

   3)17650
   ————————
  Ans. 5883⅓cts. *or* $58 83⅓cts.

quotient in dollars ; and annexing other cyphers and dividing by 3, will give tenths, hundredths, &c. of a dollar ; or dimes, cents, &c.

3. How many dollars in £.41 14s. ?

Ans. $139.

4. In £.605 11s. how many dollars ?

Ans. $2018 50 cts.

5. In £.26 1s. how many dollars and cents ?

Ans. $86 83⅓ cts.

6. How many dollars in £.1 17s. ?   Ans. $6 16⅔ cts.

CASE III. If there are shillings, pence, &c. in the given sum, annex for the shillings as before, and to these add the farthings contained in the pence and farthings ; observing to increase their number by 1 when they exceed 12, and by 2 when they exceed 36 ; and divide as before.

EXAMPLES.

1. Reduce £.34 16s. 4½d. to Federal Money.

3)34819

$.   cts.

Ans. 11606⅓ cts.   Or 116 06⅓.

2. Reduce £.108 13s. 2¼d. to Federal Money.

3)108659

$.   cts.

Ans. 36219⅔ cts.   Or 362 19⅔

3. How many dollars and cents in £.29 11s. 10d. ?

Ans. $98 64 cts.

4. In £.2001 1s. 3½d. how many dollars ?

Ans. $6670 21⅔ cts.

5. In £.24 11s. 7¾d. how many dollars and cents ?

Ans. 8194 cts. Or $81 94 cts.

6. In £.591 11s. 9½d. how many dollars?

Ans. $1971 96⅔ cts.

To reduce FEDERAL MONEY *to New England currency.*

CASE I. When the sum is dollars only, multiply by 3 ; and double the product of the first figure for shillings, and the rest of the product will be pounds.

EXAMPLES.

1. Reduce 473 dollars to New England currency.

473

3

Ans. £.141 18s.

D

2. How many pounds, &c. in 579 dollars?

                   Ans. £.173 14s.

3. In 474 dollars, how many pounds?

                   Ans. £.142 4s.

4. In 240 dollars, how many pounds?

                   Ans. £.72.

CASE. II. When there are cents in the given sum, mul-tiply the whole by 3, and cut off three figures of the pro-duct to the right hand as a remainder; multiply the re-mainder by 20, and cut off as before; proceed in the same manner through the several parts of a pound, and the num-bers standing on the left hand make the answer in the seve-ral denominations.

NOTE. If there be mills, cut off four figures, and proceed as before.

EXAMPLES.

1. Reduce $376 27cts. to New England currency,

               376,27
                    3
            ————
          £.112,881
                 20
           ————
          s.17,620
               12
          ————
          d.7,440
               4
          ————
          qr.1,760

          Ans. £.112 17s. 7¼d.

2. Reduce $609 88⅓ cts. to New England currency.

                   Ans. £.182 19s. 3½d.

3. How many pounds, shillings, &c. in $429 21 *cts.*
5 *mills* ?

<div align="center">

429215

3

_____

£.128,7645

20

_____

*s.*15,2900

12

_____

*d.*3,4800

4

_____

*qr.*1,9200

</div>

Ans. £.128 15*s.* 3¼*d.*

4. How many pounds, shillings, &c. in $4128 46 *cts.*
6⅔ *mills* ?   Ans. £.1238 10*s.* 9¼*d.*

PRACTICAL QUESTIONS IN FEDERAL MONEY.

1. Having borrowed one hundred dollars, and paid at
one time seventy dollars, and at another time sixteen dollars seven cents; what is still due ?
                    Ans. $13 93 *cts.*

2. What will 25 bushels of corn come to, at 92 cents
per bushel ?   Ans. $23.

3. What will 376 pounds of butter come to, at 18 cents
per pound ?   Ans. $67 68 *cts.*

4. What will 39 bushels of wheat come to, at 1 dollar
75 cents per bushel ?   Ans. $68 25 *cts.*

5. If iron cost 6 dollars 50 cents per cwt. what is it per
pound ?   Ans. 5 *cts.* 8 *mills.*

6. If 240 pounds of pork come to 28 dollars 80 cents, what
is it per pound ?   Ans. 12 *cts.*

7. Borrowed 607 dollars 20 cents; paid £.127 16*s.* 9*d.*
what is the balance ?   Ans. $181 7½ *cts.*

8. Lent 400 dollars; received 150 bushels of wheat at
10*s.* per bushel, and 200 pounds of butter at 17½ cents per
pound, in payment; how much is still due ?
                    Ans. $115.

9. What will 55 yards of linen come to in Federal Money, at 3s. 9d. per yard, New England currency?
                              Ans. $34 37 cts. 5 mills.

10. What will 36 yards of broadcloth come to in New England currency, at 6 dollars 25 cents per yard?
                              Ans. £.67 10s.

11. What will 7 tons of hay come to, at 16 dollars 75 cents per ton?   Ans. $117 25 cts.

12. What is the price of one cwt. of hay at 16 dollars 75 cents per ton?   Ans. 83 cts. 7½ mills.

. . . . . .

COMPOUND ADDITION.

Compound Addition in Arithmetic, teaches to collect several numbers of different denominations into one sum.

Rule.* Place the numbers of the same denominations under each other. Find the sum of the figures in the lowest denomination; divide this sum by as many of the same denomination as make one of the next higher, setting down the remainder under the column added, and carry the quotient to the next higher denomination; with which proceed as before; continuing through all the denominations to the highest, which add as in simple addition. The method of proof is the same as in simple addition.

E X A M P L E S.

MONEY.

£.	s.	d.	£.	s.	d.	qr.	£.	s.	d.
4210	16	11¼	97	12	6	2	63	9	4¼
1729	17	9	19	14	5	3	81	12	7
56	11	3½	46	17	9	1	27	16	10½
207	18	6¾	22	19	10	2	16	14	6¼
14	6	0¼	57	1	2	0	41	9	5

Sum. 6219 10 6¾

2008 13 7½

Proof. 6219 10 6¾

* The general rule in compound numbers for carrying from a lower denomination to a higher, as many units of

TROY WEIGHT.

℔.	oz.	dwt.	gr.	℔.	oz.	dwt.	gr.
671	10	16	13	47	6	11	17
392	8	9	21	56	11	15	19
249	7	12	12	63	9	8	22
516	4	3	7	26	5	19	16
627	5	17	16	21	6	12	14

AVOIRDUPOIS WEIGHT.

Ton.	cwt.	qr.	℔.	oz.	dr.	Ton.	cwt.	qr.	℔.
371	12	1	20	10	13	42	19	3	26
123	14	2	15	6	7	89	10	2	13
407	9	3	12	11	9	16	8	1	17
513	13	0	24	9	15	27	14	0	22
624	15	1	17	6	11	50	3	2	15

APOTHECARIES WEIGHT.

℔.	℥.	ʒ.	Э.	gr.	℔.	℥.	ʒ.	Э.	gr.
26	10	7	2	13	17	9	4	1	14
54	7	2	1	12	55	10	6	2	10
76	8	3	0	14	61	4	2	1	9
83	9	4	2	6	92	11	5	0	18
41	6	0	1	19	21	6	3	1	17

the next higher denomination as are contained in the lower,
is evidently correct from what has been said in simple ad-
dition; for as in simple numbers, one for every ten is
carried from a lower to a higher denomination, because one
in the latter place is of the same value as ten in the for-
mer; so carrying, in compound numbers as the general
rule directs, is only arranging the sum, or value of each
column in other columns, according to the scale of denom-
inations.

COMPOUND ADDITION.

CLOTH MEASURE.

Yd.qr.na.	E.E.qr.na.	E.Fr.qr.na.	E.Fl.qr.na.
46 1 2	74 2 3	86 5 2	29 1 2
12 3 3	51 4 1	61 4 1	17 2 3
62 1 1	24 1 2	52 3 0	20 1 2
83 2 0	56 3 1	24 2 1	75 0 1
41 0 1	31 1 2	10 0 3	46 1 3

LONG MEASURE.

Deg. mi. fur. pol. ft. in. bar.	Mi. fur. rod. yd. ft.
21 17 2 26 12 10 2	62 1 36 4½ 2
46 58 6 19 14 6 1	75 3 24 3 1
18 62 4 31 10 7 0	49 5 12 2½ 2
21 19 7 14 8 3 2	14 4 17 1 0
62 37 1 29 7 6 1	25 2 10 3 2

LAND OR SQUARE MEASURE.

Acre. rood. rod. yd. ft. in.	Acre. rood. rod.
271 1 27 16 4 110	21 2 37
424 2 31 21¼ 2 96	52 1 24
512 3 16 10 7 71	28 1 16
327 2 21 9 1 120	43 3 31
146 1 12 21¼ 6 74	65 0 20

CUBIC OR SOLID MEASURE.

T. round. ft. inch.	T. hewn. ft.	Cord. ft.
21 36 476	67 30	36 102
62 17 965	24 27	61 90
47 19 816	62 14	52 76
31 28 1146	39 36	9 120
19 34 1452	17 20	12 34

DRY MEASURE.

Chal.	bus.	pk.	qt.	pt.
39	4	1	6	1
57	6	3	4	0
61	7	2	3	1
52	4	2	5	1
42	1	1	2	1

Bus.	pk.	gal.	qt.	pt.
62	2	4	3	1
79	3	0	2	1
42	2	1	1	1
17	1	1	2	0
86	0	1	0	1

WINE MEASURE.

Tun.	hhd.	gal.	qt.	pt.	gill.
2	1	42	1	0	3
6	3	51	2	1	1
9	2	60	1	1	2
7	1	26	1	0	3
5	1	17	3	1	1

Hhd.	gal.	qt.
68	31	3
47	59	1
29	48	1
68	37	0
47	18	1

ALE OR BEER MEASURE.

Hhd.	gal.	qt.
416	29	3
39	17	2
178	15	1
315	47	1
351	34	1

A. fir.	gal.	qt.
69	7	2
43	5	1
53	4	1
16	3	3
29	1	1

Butt.	bbl.	gal.	qt.
25	1	24	1
36	2	30	2
81	0	16	0
74	1	25	1
98	1	18	1

TIME.

Yr.	m.	w.	d.	h.	m.	s.
37	10	2	6	17	31	37
46	9	3	2	14	47	25
58	7	1	4	13	19	31
61	5	0	5	12	50	47
94	12	3	1	21	39	56

Yr.	days.	h.	m.
41	276	20	30
81	310	17	48
47	163	8	29
21	360	19	50
72	176	14	33

CIRCULAR MOTION.

S	⊖	'	''		⊖	'	''
2	15	42	57		41	54	39
5	19	55	33		64	27	21
1	22	47	28		52	36	42
3	10	20	12		78	19	16
1	9	11	24		93	25	34

. . . . . .

COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION teaches to find the difference of two numbers of different denominations.

RULE.* Place the less number under the greater, so that those of the same denominations may stand directly under each other. Begin at the right hand, and take the number in each denomination of the lower line from the number standing above it, and set down the remainders below them; but if the lower number be greater than the upper, increase the upper number by as many as shall make one of the next higher denomination, and from this sum take the lower number, and set down the remainder, as before. Carry the unit borrowed to the next number in the lower line, and subtract as before; and thus proceed through all the denominations.—The method of proof is the same as in simple subtraction.

EXAMPLES.

MONEY.

	£.	s.	d.		£.	s.	d. qr.		£.	s.	d.
From	691	12	6¼		34	11	4 1		81	17	6¼
Take	284	15	9¾		17	14	10 2		21	12	4½
Rem.	406	16	8¾								
Proof.	691	12	6¼								

* The reason of this rule will readily appear from what has been said in simple subtraction; for the borrowing depends upon the same principle; and is only different, as the numbers to be subtracted are of different denominations.

TROY WEIGHT.

℔.	oz.	dwt.	gr.		℔.	oz.	dwt.	gr.
39	8	14	16		71	9	16	11
16	10	10	19		35	1	17	20

AVOIRDUPOIS WEIGHT.

Ton.	cwt.	qr.	℔.		Cwt.	qr.	℔.	oz.	dr.
73	11	1	21		94	2	11	8	9
39	17	2	12		47	3	17	11	10

APOTHECARIES WEIGHT.

℔.	℥.	ℨ.	Ə.	gr.		℔.	oz.	dr.	sc.	gr.
81	7	2	0	11		69	10	4	1	12
47	2	4	2	15		30	11	1	2	17

CLOTH MEASURE.

Yd.	qr.	na.		E.Fl.	qr.	na.		E.E.	qr.	na.		E.Fr.	qr.	na.
42	1	2		74	1	3		21	3	1		89	4	2
17	2	1		40	2	1		9	4	2		12	0	3

LONG MEASURE.

Deg.	mi.	fur.	pol.	ft.	in.	bar.		Mi	fur.	pol.	yd.	ft.
81	30	4	24	12	6	1		74	3	16	4	1
29	41	5	13	14	1	2		48	5	37	2	2

LAND OR SQUARE MEASURE.

Acre.	roo.	per.	yd.	inch.		Ac.	roo.	per.
93	2	27	14	101		39	2	17
64	3	14	16¼	97		16	3	19

Cubic or Solid Measure.

T.rou. ft. inches.	T.hewn. ft.	Cord. ft.
64 37 1141	802 26	31 39
17 19 1400	204 31	9 107

Dry Measure.

Chal. bus. pk.	Bus. pk. gal. qt. pt.
103 17 1	62 1 0 2 1
94 31 2	15 2 0 3 1

Wine Measure.

Tun. hhd. gal. qt.	Hhd. gal. qt. pt. gill.
320 1 28 1	61 31 2 1 1
91 2 45 4	17 51 3 0 1

Time.

Yr. m. w. d. h. m. s.	Yr. days. h. m.
1807 5 3 6 5 40 21	17 231 11 26
912 11 1 4 11 51 36	9 324 4 47

. . . . . .

COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION teaches to find the amount of any given number of different denominations by repeating it any proposed number of times.

RULE.* Place the multiplier under the lowest denomination of the multiplicand. Multiply the number of the low-

---

* The reason of this rule may be seen from what has been said in simple multiplication and in compound addition.

est denomination by the multiplier, and find how many of the next higher denomination are contained in the product ; write down the excess, and carry the quotient to the product of the next higher denomination, with which proceed as before, through all the denominations to the highest ; whose product, with the several excesses, will be the whole amount required.—The method of proof is the same as in simple multiplication.

CASE I. EXAMPLES.

	1.	2.	3.
	£. s. d.	£. s. d. qr.	£. s. d.
Multiply	37 12 6¾	21 17 9 2	11 9 4¼
by	5	4	6

Prod. £.188 2 9¾

4. What will 7 barrels of flour come to, at £.2 16s. 6d. per barrel ?   Ans. £.19 15s. 6d.

5. What is the weight of 6 barrels of sugar, each 1 cwt. 3 qrs. 20℔. ?   Ans. 11cwt. 2qrs. 8℔.

6. What is the weight of 12 hogsheads of sugar, each 13 cwt. 2 qrs. 26℔. ?   Ans. 164 cwt. 3 qrs. 4℔.

7. What is the number of yards in 8 pieces broadcloth, each 32 yds. 2 qrs. 1 na.   Ans. 260 yds. 2 qrs.

8. What will 6 pounds tea come to, at 5s. 3d.
                    Ans. £.1 11s. 6d.

9. . , . 8 pounds coffee, at 2s. 1¼d. . . . 17

10. . . . . 11 gallons rum, at 5s. 9¾d. . . 3 3 11¼

11. . . . . 5 bushels of wheat, 11s. 3d. . 2 16 3

12. . . . 7 yards broadcloth, 27s. 6d.   9 12 6

13. . . . 3 gallons brandy. 6s. 7¼d. . . 19 9¾

14. . . . 12 cwt. flour, 28s. 9d. 17 5 0

CASE II. If the multiplier exceed 12, multiply successively by its component parts, as in Case II in simple multiplication.

EXAMPLES.

1. What will 21 yards of calico come to at 2s. 7½d. per yard?

                    2 7½
        3×7=21   7
                    ─────
                    18 4½ price of 7 yards.
                    3
                    ─────
        Ans, £.2 15 1½ price of 21 yards.

2. What will 18 yards come to, at £.1 7s. 2½d. per yard?

Ans. £.24 9s. 9d.

3. . . . 36 pair shoes, . . 13s. 4d. £.24 00 0
4. . . . 49 yds. broadcloth, . 17s. 6¾d. 43 0 6½
5. . . . 110 pounds coffee, . 2s. 3¼d. 12 9 9¼
6. . . . 42 gallons N. E. rum, 3s. 4d. . 7 0 0
7. . . . 121 bushels corn, . . 4s. 3d. . 25 14 3
8. . . . 16 yards linen, . . 4s. 9d. . 3 16 0
9. . . . 28 bushels rye, . . 6s. 5¾d. 9 1 5
10. . . . 144 pounds indigo, . 16s. 6¼d. 119 2 0

CASE III. If the multiplier be not a composite number, find the nearest to it, either greater or less; multiply by the component parts as before, and for the odd parts add or subtract as the case requires.

EXAMPLES.

1. What will 65¼ yards of cloth come to, at £.1 14s. 6⅔d. per yard?

$$£.1 \quad 14 \quad 6\tfrac{1}{2}$$
$$8$$
_____

8×8=64   13 16 4 price of 8 yards.
64+1¼=65¼   8
_____

110 10 8 price of 64 yards.
4)   1 14 6¾ price of 1 yard.
8 7½ price of ¼ yard.
_____

Ans. £.112 13 10

2. What will 76 yards cost, at 14s. 9½d.?

$$£.0 \quad 14 \quad 9\tfrac{1}{2}$$
7×11=77—1=76   11
_____

8 2 8½
7
_____

56 18 11½ price of 77 yards.
Subtract   14 9½ price of 1 yard.
_____

Ans. £.56 4 2

3. What will 51 pounds tea cost, at 3s. 6d. per pound ?
                       Ans. £.8 18s. 6d.

4. What will 183 gallons brandy cost, at 7s. 5d. ?

    $10 \times 10 = 100$

      $10 \times 8 = 80$

          $3 = 3$ Then $100 + 80 + 3 = 183$

                           Ans. £.67 17s. 3d.

5. . . . 57½ pounds tea, at 4s. 2½d. £.12 1s. 11¾d.

6. . . . 43 yards, . . £.1 6s. 8⅓d.   57 8s. 5¼d.

7. . . . 145 pounds chocolate, 2s. 6d. 18 2s. 6d.

8. . . . 600 yards cloth, £.1 2s. 7½d. 678 15s. 0d.

    $10 \times 10 \times 6 = 600$

             . . . . .

COMPOUND DIVISION.

COMPOUND DIVISION teaches to find how often one number is contained in another of different denominations.

RULE.* Place the numbers as in simple division. Begin at the left hand, and divide each denomination by the divisor, setting the quotients under their respective dividends ; but if there be a remainder in dividing any of the denominations except the lowest, find how many of the next lower denomination it is equal to ; and add it to the number, if any, which was in this denomination before ; divide this sum as usual, and thus proceed, until the whole is finished.

The method of proof is the same as in simple division.

CASE I. EXAMPLES.

	1.			2.				3.	
£.	s.	d.	£.	s.	d.	qr.	£.	s.	d.
8)39	11	6¼	6)22	9	4	2	5)71	13	8¾
4	18	11¼							

* To divide a number consisting of several denominations by a simple number, is evidently the same as dividing all the parts of which it is composed, by the same simple number. And this will be true, when any of the parts are not an exact multiple of the divisor ; for if the excess of that multiple has its proper value in the next lower denomination, the dividend will still be divided into parts, and the true quotient be found as before.

E

4. Divide £.67 10s. 9d. by 7, . . Ans. £.9. 12 11$\frac{1}{2}$.

5. . . £.248 9s. 1$\frac{1}{2}$d. by 9, . . Ans. £.27 12 1$\frac{1}{2}$.

6. . . £.4 10s. 3d. by 12, . . Ans. · 7 6$\frac{1}{4}$.

7. . . £.13 0s. 3d. by 6, . . Ans. £.2 · 3 4$\frac{1}{2}$.

8. . . . 13 cwt. 1 qr. 12 ℔. 6 oz. 10 dr. by 11.

Ans. 1 cwt. 0 qr. 24 ℔. 0 oz. 9$\frac{7}{11}$ dr.

9. . ! . 17 cwt. 1 qr. into 6 equal parts.

Ans. 2 cwt. 3 qrs. 14 ℔.

10. . . . £.8 equally among 6 persons.

Ans. £.1 6s. 8d.

CASE II. If the divisor exceed 12, divide continually by its component parts, as in simple division, Case II.

EXAMPLES.

1. Divide £.37 16s. equally among 24 men.

£. s. d.
6)37 16 0
_____
4)6 6 0
_____
£.1 11 6 Ans.

2. Divide a hogshead of sugar, weighing 12 cwt. 2 qrs. 26 ℔. equally among 16 men.

Cwt. qr. ℔. oz.
4)12 2 26 0
_____
4)3 · 0 20 8
_____
3 5 2

Ans. 3 qrs. 5 ℔. 2 oz. each.

3. If 20 gallons brandy cost £.7 5s. 10d. what is it per gallon? Ans. 7s. 3$\frac{1}{2}$d.

4. If £.165 be divided equally among 99 persons, what will each have ? Ans. £.1 13s. 4d.

5. If 72 gallons of rum come to £.20 9s. 6d. what is it per gallon ? Ans. 5s. 8d. 1 qr.

6. If 108 ℔ tea cost £.45 13s. 6d. what is one pound worth ? Ans. 8s. 5$\frac{1}{2}$d.

CASE III. If the divisor be not a composite number, divide as in long division.

EXAMPLES.

1. Divide £.391 17s. 6¼d. by 46.

```
        £.  s.  d.  £.  s.  d.
    46)391 17  6¼(8 10 4½ Ans.
       368
       ───
        23
        20
       ───
       46)477(10s.
          46
          ───
          17
          12
          ───
          46)210(4d.
             184
             ───
              26
               4
             ───
             46)105(2 qrs.
                92
                ──
                13
```

2. If 263 bushels wheat cost £.86 11s. 5d. what is it p
bushel ?

```
              £.  s.  d.
          263)86 11  5(0 6s. 7d. Ans.
                 20
              ───
              263)1731(6s.
                  1578
                  ────
                   153
                    12
                  ────
                  263)1841(7d.
                      1841
```

3. Divide 27 ton, 13 cwt. 2 qrs. by 34.

```
            T.  cwt.  qr.  T.  cwt.  qr.  ℔.
        34)27   13    2 ( 0   16    1    3
           20

        34)553(16        Ans. 16 cwt. 1 qr. 3 ℔.
           34

           213
           204

             9
             4

        34)38(1
           34

            4
           28

        34)112(3
           102

            10
```

4. If 46 ℔ indigo cost £.53 10s. 6d. what is it per pound ?
Ans. £.1 3s. 3¼d.

5. If 59 yards of cloth cost £.8 17s. what is it per yard ?
Ans. 3s.

6. If 109 ℔ of coffee cost £.8 12s. 7d. what is it per pound ?   Ans. 1s. 7d.

7. If 79 ℔ of tea cost £.32 11s. 9d. what is it per pound ?
Ans. 8s. 3d.

8. If 263 bushels of wheat cost $287 97 cts. 3 mills, what is it per bushel ?   Ans. $1 9 cts. 4 mills.

9. If 397 yards of cloth cost £.66 3s. 4d. how much is it per yard ?   Ans. 3s. 4d.

10. If 37 thousand of boards come to $203 50 cts. what is one thousand worth ?   Ans, $5 50 cts.

DUODECIMALS.

DUODECIMALS chiefly regard feet and inches. They are
so called, because they decrease by twelves from the place
of feet towards the right hand.

Inches are sometimes called primes, and marked thus
(′) ; the next division is called parts or seconds, and mark-
ed (″) ; the next thirds, and marked (‴) ; &c.

MULTIPLICATION OF DUODECIMALS.

RULE. Under the multiplicand write the corresponding
denominations of the multiplier. Multiply each term in
the multiplicand, beginning at the lowest, by the highest
denomination in the multiplier ; and write each result un-
der its respective term ; observing to carry an unit for ev-
ery 12 from each lower place to its next higher.

In the same manner multiply all the multiplicand by the
next highest denomination in the multiplier ; and set the
result of each term removed one place to the right hand of
those in the multiplicand.

Proceed in like manner with the remaining denomina-
tions, and the sum of all the lines will be the product re-
quired.

EXAMPLES.

1. Multiply 4 feet 2 inches by 3 feet 5 inches.

```
        Ft.  ′
         4   2
         3   5
       ─────────
        12   6   ″
         1   8  10
       ─────────────
        14   2  10   Ans. 14 ft. 2′ 10″
       ─────────────
```

2. Multiply 10 feet 11 inches by 7 inches.

```
        10   11
              7
       ─────────
         6   4   5   Ans. 6 feet 4′ 5″
       ─────────
```

E 2

3. What is the content of a bale 6 feet 5′ long ; 4 feet 3′ high, and 3 feet 10′ wide ?

```
            Ft.  ′
             6   5
             4   3
           ─────────
            25   8   ″
             1   7   3
           ─────────
            27   3   3
             3  10
           ─────────
            81   9   9   ‴
            22   8   8   6
           ─────────
           104   6   5   6   Ans. 104 ft. 6′ 5″ 6‴
```

4. How many square feet in a board 25 feet 6 inches long, and 1 foot 3 inches wide ? Ans. 31 feet 10¼ inches.

5. How many cubic feet in a stick of timber 12 feet 10′ long, 1 foot 7′ wide, and 1 foot 9′ thick ?
                            Ans. 35 ft. 6′ 8″ 6‴

6. How many cubic feet of wood in a load 7 feet 10′ long, 3 feet 11′ wide, and 3 feet 6′ high ?
                            Ans. 107 ft. 4′ 7″

. . . . .

VULGAR FRACTIONS.

FRACTIONS are expressions for any assignable parts of an unit,* and are represented by two numbers, placed one above another, with a line drawn between them.

The number above the line is called the *numerator*, and that below the line the *denominator*.

---

* Unity or one, is the least number which is the subject of the rules in common arithmetic ; but it is necessary on many occasions to consider arithmetical quantities less than one ; to suppose unity to be broken into many equal parts, and to make a certain number of those equal parts the subject of consideration. Such a number of equal parts is called *a fraction*.

_

The denominator shows into how many equal parts the unit or integer is divided, and the numerator shows how many of those equal parts are denoted by the fraction.

Fractions are either proper, improper, single, compound or mixed.

1. A *proper fraction* is when the numerator is less than the denominator : As, $\frac{3}{4}$, $\frac{4}{7}$, $\frac{5}{9}$, &c.

2. An *improper fraction* is when the numerator exceeds the denominator : As $\frac{8}{3}$, $\frac{110}{21}$, &c.

3. A *single fraction* is a simple expression for any number of parts of the integer.

4. A *compound fraction* is the fraction of a fraction : As $\frac{1}{2}$ of $\frac{4}{5}$, $\frac{3}{4}$ of $\frac{7}{8}$, &c.

5. A *mixed number* is composed of a whole number and a fraction : As $8\frac{1}{5}$, $12\frac{9}{13}$, &c.

NOTE. Any number may be expressed like a fraction by writing 1 under it : Thus, $\frac{6}{1}$ means 6 ones or 6.

6. The *common measure* of two or more numbers is that number which will divide each of them without a remainder ; and the *greatest* number that will do this, is called the *greatest common measure*.

7. A number, which can be measured by two or more numbers, is called their *common multiple ;* and if it be the *least* number, which can be so measured, it is called their *least common multiple.**

<div align="center">PROBLEM 1.</div>

To find the greatest common measure of two or more numbers.

RULE.† 1. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and

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* A *prime number* is that, which can only be measured or divided into equal parts by an unit ; as 3, 5, 7, &c.

That number which is produced by multiplying several numbers together, is called a *composite number*.

A perfect number is equal to the sum of all its aliquot parts.

† The truth of this rule may be shown from the first example. For since 54 measures 108, it also measures 108 +54 or 162. Again, since 54 measures 108 and 162, it also measures $5\times162+108$ or 918. In the same manner it will be found to measure $2\times918+162$ or 1998 ; and so

so on ; always dividing the last divisor by the last remain-
der, until nothing remains ; then will the last divisor be the
greatest common measure required.

2. When there are more than two numbers, find the
greatest common measure of two of them, as before ; and
next find the greatest common measure of that common
measure and one of the other numbers ; and so on, through
all the numbers to the last ; then will the greatest com-
mon measure last found be the answer.

3. If one happen to be the common measure, the given
numbers are prime to each other, and found to be incom-
mensurable.

<div align="center">EXAMPLES.</div>

1. Required the greatest common measure of 918, 1998,
and 522.

```
      918)1998(2                 So 54 is the greatest common
           1836                  measure of 1998 and 918—
           ----
        162)918(5                Hence 54)522(9
            810                            486

          108)162(1                       36)54(1
              108                            36
              ----                          --
            54)108(2                      18)36(2
               108                           36
               ---                           --
```

Therefore 18 is the answer required.

2. What is the greatest common measure of 612 and
540 ?   Ans. 36.

3. What is the greatest common measure of 117 and 91 ?
                                             Ans. 13.

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on.   Therefore 54 measures both 918 and 1998. It is
also the greatest common measure ; for suppose there be
a greater, then, since the greater measures 918 and 1998,
it also measures the remainder, 162 ; and since it measures
162 and 918, it also measures the remainder 108. In the
same manner it will be found to measure the remainder
54 ; that is, the greater measures the less, which is absurd.
Therefore 54 is the greatest common measure.

In the same manner, the demonstration may be applied
to 3 or more numbers.

PROBLEM 2.

To find the least common multiple of two or more numbers.

RULE.* 1. Divide by any number, that will divide two or more of the given numbers without a remainder, and set the quotients together with the undivided numbers, in a line beneath.

2. Divide the second line as before, and so on, until there are no two numbers that can be divided; then the continued product of the divisors and quotients will give the multiple required.

EXAMPLES,

1, What is the least common multiple of 3, 5, 8 and 10?

     2)3 5 8 10
        ————————————
       5)3 5 4   5
        ————————————
        3 1 4   1   Then $2 \times 5 \times 3 \times 4 = 120$ the answer.
        ————————————

2. What is the least common multiple of 9, 8, 15, 16 ?
                                    Ans. 720.

3. What is the least number that 3, 4, 8 and 12 will measure ?   Ans. 24.

4. What is the least number that can be divided by the 9 digits without a remainder?   Ans. 2520.

REDUCTION OF VULGAR FRACTIONS.

REDUCTION OF VULGAR FRACTIONS is the bringing them out of one form into another, in order to prepare them for the operations of addition, subtraction, &c.

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* The reason of this rule may be shown from the first example, thus : It is evident, that $3 \times 5 \times 8 \times 10 = 1200$ may be divided by 3, 5, 8 and 10, without a remainder ; but 10 is a multiple of 5, therefore $3 \times 5 \times 8 \times 2$, or 240, is also divisible by 3, 5, 8 and 10. Also 8 is a multiple of 2 ; therefore $3 \times 5 \times 4 \times 2 = 120$ is also divisible by 3, 5, 8 and 10 ; and is evidently the least number that can be so divided.

Case * I.

To abbreviate or reduce fractions to their lowest terms.

Rule.† Divide the terms of the given fraction by any number that will divide them without a remainder, and

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* All the operations of fractions depend on these two maxims. 1. If the numerator of a fraction be increased while the denominator continues the same, the value of the fraction will be increased proportionably; and the reverse. 2. If the denominator be increased in any proportion, while the numerator continues the same, the value of the fraction will be diminished in a contrary proportion, and the reverse.

These maxims will appear plain, on considering, that every fraction is equivalent to the quotient of the numerator divided by the denominator.

From these two maxims it follows, that if the numerator and denominator of a fraction, be both multiplied, or both divided by the same number, the value of the fraction will not be affected.

† It has already been shown, that dividing both the terms of a fraction by the same number, will give another fraction equal to the former, but in lower terms; and it is evident, that if these divisions are performed as often as can be done, or if the common divisor be the greatest possible, the terms of the resulting fractions must be the least possible.

To render these divisions easy, we have the following rules.

1. Any number ending with an even number or a cypher, is divisible by 2.

2. Any number ending with 5, or 0, is divisible by 5.

3. If the right hand place of any number be 0, the whole is divisible by 10.

4. If the two right hand figures of any number are divisible by 4, the whole is divisible by 4.

5. If the sum of the digits constituting any number be divisible by 3, or 9, the whole is divisible by 3, or 9.

6. All prime numbers, except 2 and 5, have 1, 3, 7 or 9 in the place of units; and consequently all other numbers are composite and capable of being divided.

7. When numbers, with the sign of addition or subtraction between them, are to be divided by any number, each

these quotients again in the same manner ; and so on until it appears that there is no number greater than 1, which will divide them ; and the fraction will then be in its lowest terms :—Or, divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

EXAMPLES.

1. Reduce $\frac{144}{240}$ to its lowest terms.

$$\overset{(4)}{} \quad \overset{(3)}{} \quad \overset{(4)}{}$$
$$\frac{144}{240}=\frac{36}{60}=\frac{12}{20}=\frac{3}{5}, \text{ the answer.}$$

Or thus. 144)240(1
144

96)144(1
96

48)96(2
96

Therefore 48 is the greatest common measure, and $48)\frac{144}{240}=\frac{3}{5}$ Ans.

2. Reduce $\frac{48}{272}$ to its lowest terms.   Ans. $\frac{3}{17}$.
3. Reduce $\frac{825}{960}$ to its lowest terms.   Ans. $\frac{55}{64}$.
4. Reduce $\frac{60}{135}$ to its lowest terms.   Ans. $\frac{12}{27}$.
5. Reduce $\frac{498}{118}$ to its lowest terms.   Ans. $\frac{249}{592}$.
6. Reduce $\frac{3619}{6231}$ to its lowest terms.   Ans. $\frac{11}{19}$.

CASE II. *To reduce a mixed number to its equivalent improper fraction.*

RULE.* Multiply the whole number by the denominator of the fraction, and add the numerator to the product ;

of the numbers must be divided. Thus $\frac{4+8+10}{2}=2+4+5=11$.

8. But if the numbers have the sign of multiplication between them, only one of them must be divided. Thus $\frac{3\times8\times10}{2\times6}=\frac{3\times4\times10}{1\times6}=\frac{1\times4\times10}{1\times2}=\frac{1\times2\times10}{1\times1}=\frac{20}{1}=20$.

* The truth of this rule will appear on considering, that every integer or unit, is equal in value to so many parts as it is broken into ; that is, to so many fractional parts as are expressed by the denominator of the fraction.

then that sum written above the denominator will form the fraction required.

EXAMPLES.

1. Reduce $27\frac{2}{9}$ to its equivalent improper fraction.

$$
\begin{array}{r}
27 \\
9 \\
\hline
243 \\
2 \\
\hline
245 \\
\hline
9
\end{array}
$$

     Or $\dfrac{\overline{27\times9}+2}{9}=\frac{245}{9}$ the answer.

2. Reduce $514\frac{5}{16}$ to an improper fraction.   Ans. $\frac{8229}{16}$.

3. Reduce $12\frac{15}{17}$ to an improper fraction.   Ans. $\frac{219}{17}$.

4. Reduce $79\frac{2}{19}$ to an improper fraction.   Ans. $\frac{1513}{19}$.

5. Reduce $100\frac{19}{59}$ to an improper fraction.   Ans. $\frac{5919}{59}$.

CASE III.

To reduce an improper fraction to its equivalent whole or mixed number.

RULE.* Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

EXAMPLES.

1. Reduce $\frac{981}{16}$ to its equivalent whole, or mixed number.

$$
\begin{array}{r}
16)981(61\frac{5}{16} \\
96 \\
\hline
21 \\
16 \\
\hline
5
\end{array}
$$

     Or $\frac{981}{16}=981\div16=61\frac{5}{16}$ the answer.

2. Reduce $\frac{219}{17}$ to its equivalent whole or mixed number.

                   Ans. $12\frac{15}{17}$.

3. Reduce $\frac{126}{48}$ to its equivalent whole or mixed number.

                    Ans. $2\frac{5}{8}$.

---

* This rule is plainly the reverse of the former, and has its reason in the nature of common division.

4. Reduce $\frac{5}{9}$ to its equivalent whole or mixed number.
<div style="text-align:right">Ans. 7.</div>

5. Reduce $\frac{621613}{514}$ to its proper terms. Ans. $1209\frac{187}{514}$.

CASE IV.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.* Multiply the whole number by the given denominator, and place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 7 to a fraction, whose denominator shall be 9.
<div style="text-align:center">$7 \times 9 = 63$, and $\frac{63}{9}$ answer.</div>
<div style="text-align:center">And $\frac{63}{9} = 63 \div 9 = 7$ proof.</div>

2. Reduce 13 to a fraction, whose denominator shall be 12.
<div style="text-align:right">Ans. $\frac{156}{12}$.</div>

3. Reduce 746 to a fraction, whose denominator shall be 60.
<div style="text-align:right">Ans. $\frac{44760}{60}$.</div>

CASE V.

To reduce a compound fraction to an equivalent single one.

RULE. † Multiply all the numerators together for the numerator, and all the denominators together for the denominator, and they will form the fraction required.

If part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction by one of the former cases.

When it can be done, divide any two terms of the fraction by the same number, and use the quotients instead thereof.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{8}{11}$ to a single fraction.

$\frac{2 \times 3 \times 4 \times 8}{3 \times 4 \times 5 \times 11} = \frac{192}{660} = \frac{16}{55}$ the answer.—Or by expunging equal numerators and equal denominators, the answer will be as before, $= \frac{16}{55}$.

---

* Multiplication and division are here equally used in the upper and lower part of the fraction, and consequently the result is the same as the quantity first proposed.

† That a compound fraction may be represented by a single one is evident, since a part of a part must be equal to

2. Reduce ⅔ of ¾ of ⅘ to a single fraction.   Ans. ¼.

3. Reduce ¹¹⁄₁₂ of ₇⁄₁₃ of ₈⁄₁₉ of 10 to a single fraction.
                                                            Ans. ¹⁵⁴⁰⁄₇₄₁.

4. Reduce ½ of ₈⁄₉ of ₆⁄₇ to a single fraction.   Ans. ₈⁄₂₁.

CASE VI.

*To reduce fractions of different denominators to equivalent
        fractions, having a common denominator.*

RULE.* Multiply each numerator into all the denominators except its own, for a new numerator ; and all the denominators continually for the common denominator.

EXAMPLES.

1. Reduce ½, ⅔ and ₄⁄₇ to equivalent fractions, having a common denominator.

        1×5×7=35 the new numerator for ½.
        3×2×7=42 . . . do. . . ⅔.
        4×2×5=40 . . . do. . . ₄⁄₇.
        2×5×7=70 the common denominator.

Therefore the equivalent fractions are ³⁵⁄₇₀, ⁴²⁄₇₀ and ⁴⁰⁄₇₀, the answer.

2. Reduce ₆⁄₁₀, ₄⁄₉, ½ and ₆⁄₇ to equivalent fractions, having a common denominator.   Ans. ³⁷⁸⁄₆₃₀, ³¹⁵⁄₆₃₀, ⁷⁰⁄₆₃₀ and ⁵⁴⁰⁄₆₃₀.

3. Reduce ⅓, ¾ of ₄⁄₅, 5½ and ₂⁄₁₃ to a common denominator.
                                        Ans. ¹⁹⁰⁄₅₇₀, ³⁴²⁄₅₇₀, ³¹³⁵⁄₅₇₀, ⁶⁰⁄₅₇₀.

4. Reduce ¹¹⁄₁₃, ¾ of 1¼, ₉⁄₁₁ and ₅⁄₇ to a common denominator.   Ans. ¹³⁵⁵²⁄₁₆₀₁₆, ¹⁵⁰¹⁵⁄₁₆₀₁₆, ¹³¹⁰⁴⁄₁₆₀₁₆, ¹¹⁴⁴⁰⁄₁₆₀₁₆.

some part of the whole. The truth of the rule for this reduction may be shown as follows.

Let the compound fraction to be reduced be ⅔ of ₄⁄₇. Then ⅓ of ₄⁄₇=₄⁄₇÷3=₄⁄₂₁, and consequently ⅔ of ₄⁄₇=₄⁄₂₁×2=₈⁄₂₁ the same as by the rule, and the like will be found to be true in all cases.

If the compound fraction consist of more numbers than 2, the two first may be reduced to one, and that one and the third will be the same as a fraction of two numbers ; and so on.

-* The result of an operation by this rule is, that the numerator and denominator of each fraction is multiplied by the same numbers.; and consequently their value is not altered, as may be seen from what has been said under the first Case.

CASE VII.

To find the value of a fraction in any known parts of the integer.

RULE.* Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator ; and if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before ; and so on, as far as necessary ; and the quotients placed after one another, in their order, will be the answer required.

EXAMPLES.

1. What is the value of $\frac{5}{7}$ of a shilling ?

$$\begin{array}{r} 5 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 7)60(8d. \\ 56 \\ \hline 4 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 7)16(2\frac{2}{7}\,qrs. \\ 14 \\ \hline 2 \end{array}$$ Ans. 8d. $2\frac{2}{7}$qrs.

2. What is the value of $\frac{5}{12}$ of a dollar ?

Ans. 41 cts. $6\frac{2}{3}$ mills.

3. What is the value of $\frac{12}{16}$ of a pound Troy ?

Ans. 9 oz.

4. What is the value of $\frac{7}{9}$ of an cwt. ?

Ans. 3 qrs. 3 ℔. 1 oz. $12\frac{4}{9}$ drs.

5. What is the value of $\frac{4}{9}$ of a mile ?

Ans. 4 fur. 22 pol. 4 yds. $2\frac{1}{7}$ ft.

6. What is the value of $\frac{4}{9}$ of a month ?

Ans. 3 w. 1 d. 9 h. 36 m.

7. What is the value of $\frac{7}{16}$ of an acre ?

Ans. 1 rood 30 poles.

---

* The numerator of a fraction may be considered as a remainder, and the denominator as a divisor ; therefore this rule has its reason in the nature of compound division.

CASE VIII.

To reduce a fraction of one denomination to that of another,
retaining the same value.

RULE.* Make a compound fraction of it, and reduce it
to a single one.

EXAMPLES.

1. Reduce $\frac{5}{6}$ of a penny to the fraction of a pound.

$$\frac{5}{6} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20} = \frac{5}{1440} = \frac{1}{288} \text{ the answer.}$$

$$\text{And } \frac{1}{288} \text{ of } \frac{20}{1} \text{ of } \frac{12}{1} = \frac{240}{288} = \frac{5}{6}\text{d. the proof.}$$

2. Reduce $\frac{1}{2}$ of a farthing to the fraction of a pound.

Ans. $\frac{1}{1920}$.

3. Reduce $\frac{4}{11}$ of a mill to the fraction of a dollar.

Ans. $\frac{1}{2750}$.

4. Reduce $\frac{1}{13}$G. to the fraction of a penny.

Ans. $\frac{40}{3} = 13\frac{1}{3}$d.

5. Reduce $\frac{5}{7}$ of a pound Avoirdupois to the fraction of
an cwt.

Ans. $\frac{6}{784} = \frac{3}{392}$.

6. Reduce $\frac{3}{13}$ of a month to the fraction of a day.

Ans. $\frac{84}{13} = 6\frac{6}{13}$.

7. † Reduce 7s. 3d. to the fraction of a pound.

Ans. $\frac{29}{80}$.

8. Reduce 6 furlongs 16 poles to the fraction of a mile.

Ans. $\frac{4}{5}$.

ADDITION OF VULGAR FRACTIONS.

RULE. ‡ Reduce compound fractions to single ones ;
mixed numbers to improper fractions ; fractions of differ- ·

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* The reason of this is explained in the rule for reducing
compound fractions to a single one.

The rule might have been distributed into two or three
different cases, but the directions here given may easily be
applied to any question that can be proposed in those cases,
and will be more easily understood by an example or two,
than by words.

† Thus 7s. 3d.=87d. and £.1=240 .. $\frac{87}{240} = \frac{29}{80}$ the an-
swer.

‡ Fractions, before they are reduced to a common de-
nominator, are entirely dissimilar, and therefore cannot be
incorporated with one another ; but when they are reduced

ent integers to those of the same ; and all of them to a
common denominator ; then the sum of the numerators,
written over the common denominator, will be the fraction
required.

EXAMPLES.

1. Add $3\frac{5}{8}$, $\frac{7}{8}$, $\frac{4}{5}$ of $\frac{7}{8}$ and 7 together.

First, $3\frac{5}{8}=\frac{29}{8}$, $\frac{4}{5}$ of $\frac{7}{8}=\frac{28}{40}=\frac{7}{10}$, $7=\frac{7}{1}$.

Then the fractions are $\frac{29}{8}$, $\frac{7}{8}$, $\frac{7}{10}$ and $\frac{7}{1}$.

$$29\times8\times10\times1=2320$$
$$7\times8\times10\times1= 560$$
$$7\times8\times8\times1 = 448$$
$$7\times8\times8\times10=4480$$

$$8\times8\times10\times1=\frac{7808}{640}=12\frac{128}{640}=12\frac{1}{5} \text{ Ans.}$$

2. Add $\frac{5}{8}$, $7\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{3}{4}$ together.   Ans. $8\frac{3}{8}$.

3. What is the sum of $\frac{1}{3}$ of 95 and $\frac{7}{8}$ of 14 ?

Ans. $43\frac{11}{12}$.

4. Add 19, 7, and $\frac{1}{2}$ of $\frac{2}{3}$ together,   Ans. $26\frac{1}{3}$.

5. Add $\frac{2}{3}$ and $17\frac{1}{2}$ together.   Ans. $18\frac{1}{6}$.

6. What is the sum of $\frac{1}{16}$. $\frac{2}{3}$s. and $\frac{5}{12}$ of a penny ?

Ans. $\frac{3139}{1008}$s. or 3s. 1d. $1\frac{10}{21}$qrs.

7. Add $\frac{2}{3}$ of $15\textit{s}$. $3\frac{3}{4}\textit{s}$. $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{2}{3}$ of a pound and $\frac{2}{3}$ of $\frac{3}{4}$ of
a shilling together.   Ans. $7\textit{s}$. 17s. $5\frac{3}{4}$d.

8. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot and $\frac{3}{8}$ of a mile together.

Ans. 660 yds. 2 ft. 9 in.

9. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day and $\frac{1}{2}$ of an hour together.

Ans. 2 days, $14\frac{1}{2}$ hours.

10. Add $\frac{4}{7}$ of a ton to $\frac{9}{10}$ of an hundred weight.

Ans. 12 cwt. 1 qr. 8 ℔. 12 oz. $12\frac{4}{5}$ drs.

to a common denominator, and made parts of the same thing,
their sum or difference may then be as properly expressed
by the sum or difference of the numerators, as the sum or
difference of any two quantities whatever, by the sum or
difference of their individuals ; whence the reason of the
rules both for addition and subtraction becomes manifest.

SUBTRACTION OF VULGAR FRACTIONS.

RULE. Prepare the fractions as in addition, and the difference of the numerators written above the common denominator, will give the difference of the fraction required.

EXAMPLES.

1. From $\frac{2}{3}$ take $\frac{2}{9}$ of $\frac{3}{7}$.

$$\frac{2}{9} \text{ of } \frac{3}{7} = \frac{2}{21} \text{ ; and } \frac{2}{3} = \frac{14}{21}.$$
$$\frac{14}{21} - \frac{2}{21} = \frac{12}{21} = \frac{4}{7} \text{ the answer.}$$

2. From $\frac{111}{112}$ take $\frac{3}{4}$.   Ans. $\frac{27}{112}$.

3. From $71\frac{1}{2}$ take $\frac{17}{19}$.   Ans. $70\frac{23}{38}$.

4. From $\frac{7}{8}\mathcal{L}$. take $\frac{3}{4}$ of a shilling.   Ans. 16s. 9d.

5. From $\frac{3}{5}$ oz. take $\frac{7}{8}$ of a pennyweight.
                            Ans. 11 dwt. 3 grs.

6. From $\frac{1}{2}$ cwt. take $\frac{7}{13}$ of a pound.
           Ans. 1 qr. 27 ℔. 6 oz. $10\frac{2}{3}$ drs.

7. From 7 weeks take 9 days $\frac{7}{10}$.
                   Ans. 5 w. 4 d. 7 h. 12 m.

8. From 4 days $7\frac{1}{2}$ hours take 1 day 9 hours $\frac{3}{13}$.
                  Ans. 2 d. 22 h. 20 m.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.* Reduce compound fractions to single ones, and mixed numbers to improper fractions; then the product of the numerators is the numerator; and the product of the denominators, the denominator of the product required.

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* Multiplication by a fraction implies the taking of some part or parts of the multiplicand; and therefore, may be truly expressed by a compound fraction. Thus $\frac{3}{4}$ multiplied by $\frac{5}{6}$ is the same as $\frac{3}{4}$ of $\frac{5}{6}$; and as the directions of the rule agree with the method already given for reducing these fractions to single ones, it is shown to be right.

EXAMPLES.

1. Required the continued product of $2\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{3}$ of $\frac{5}{6}$ and 2.

$$2\frac{1}{2}=\frac{5}{2}, \quad \frac{1}{3} \frac{5}{6}=\frac{1\times5}{3\times6}=\frac{5}{18}, \quad \text{and } 2=\frac{2}{1} \; ;$$

Then $\frac{5}{2}\times\frac{1}{8}\times\frac{5}{18}\times\frac{2}{1}=\frac{5\times1\times 5\times2}{2\times8\times18\times1}=\frac{50}{288}=\frac{25}{144}$ the answer.

2. Multiply $\frac{3}{7}$ by $\frac{3}{11}$. Ans. $\frac{9}{77}$

3. Multiply $4\frac{1}{2}$ by $\frac{1}{8}$. Ans. $\frac{9}{16}$

4. Multiply $\frac{3}{5}$ of 8 by $\frac{7}{8}$ of 5. Ans. 21.

5. Multiply $7\frac{1}{2}$ by $9\frac{1}{4}$. Ans. $69\frac{3}{8}$.

6. Multiply $4\frac{1}{2}$, $\frac{3}{4}$ of $\frac{1}{7}$, and $18\frac{4}{5}$ continually together.

Ans. $9\frac{9}{140}$.

DIVISION OF VULGAR FRACTIONS.

RULE.* Prepare the fractions as in multiplication ; then invert the divisor, and proceed as in multiplication.

EXAMPLES.

1. Divide $\frac{1}{5}$ of 19 by $\frac{2}{3}$ of $\frac{3}{4}$,

$$\frac{1}{5} \text{ of } 19=\frac{1\times19}{5\times 1}=\frac{19}{5}, \quad \text{and } \frac{2}{3} \text{ of } \frac{3}{4}=\frac{6}{12}=\frac{1}{2} \; ;$$

$$\therefore \frac{19}{5}\times\frac{2}{1}=\frac{19\times2}{5\times1}=\frac{38}{5}=7\frac{3}{5} \text{ the quotient required.}$$

2. Divide $\frac{17}{21}$ by $\frac{3}{5}$. Ans. $1\frac{22}{63}$.

---

* The reason of the rule may be shown thus : Suppose it is required to divide $\frac{3}{4}$ by $\frac{2}{5}$. Now $\frac{3}{4}\div2$ is manifestly $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{4\times2}$; but $\frac{2}{5}=\frac{1}{5}$ of 2, $\therefore \frac{1}{5}$ of 2, or $\frac{2}{5}$ must be contained 5 times as often in $\frac{3}{4}$ as 2 is ; that is $\frac{3\times5}{4\times2}=$the answer ; which is according to the rule, and will be so in all cases.

NOTE. A fraction is multiplied by an integer, by dividing the denominator by it, or multiplying the numerator. And it is divided by an integer, by dividing the numerator, or multiplying the denominator.

3. Divide 99 by 108.   Ans. $\frac{99}{108} = \frac{11}{12}$.

4. Divide $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{4}{5}$.   Ans. $1\frac{1}{4}$.

5. Divide $3\frac{1}{4}$ by $9\frac{1}{2}$.   Ans. $\frac{1}{3}$.

6. Divide $\frac{7}{8}$ by 4.   Ans. $\frac{7}{32}$.

. . . . . .

DECIMAL FRACTIONS.

A DECIMAL is a fraction whose denominator is an unit with as many cyphers annexed to it as the numerator has places; and is usually expressed by writing the numerator only, with a point before it called the separatrix. Thus $\frac{5}{10}$, $\frac{25}{100}$, $\frac{236}{1000}$ are decimal fractions, and are expressed by ,5 ,25 and ,236 respectively.

The place of a figure in decimals, as in whole numbers, determines its relative value: That in the first place next the separatrix is 10th parts; that in the second 100th parts, &c. decreasing in the same tenfold proportion to the right hand, as whole numbers increase decimally from units to the left hand.

Cyphers placed to the right hand of decimals make no alteration in their value; for ,5 ,50 ,500 &c. are decimals of the same value, being each equal to $\frac{1}{2}$; but if placed to the left hand, the value of the fraction is decreased in a tenfold proportion for every cypher prefixed; thus ,5 ,05 ,005 &c. are 5 tenth parts, 5 hundredth parts, and 5 thousandth parts respectively.

ADDITION OF DECIMALS.

RULE. Set the numbers so that the decimal points may stand directly under each other; then add as in whole numbers, and place the decimal point, in the sum, directly under the decimal points of the numbers which have been added.

EXAMPLES,

1.	2.
124,6201	3741,21
5,92	374,646
17,1174	8,46
305,2165	52117,42
2,71	91,5
455,5840	56333,236

3. What is the sum of 27,6,+39,213,+72014,9+417+5032 and +2214,298 ?   Ans. 79993,411.

4. What is the sum of ,014+,9816+,32+,15914+,72913 and +,0047 ?   Ans. 2,20857.

5. What is the sum of 27,148+918,73+14016,+294304, +7138 and +,221,7 ?   Ans. 316625,578,

SUBTRACTION OF DECIMALS.

RULE. Set the less number under the greater in the same manner as in addition ; then subtract as in whole numbers, and place the decimal point in the remainder directly under the other points.

EXAMPLES.

1.	2.	3.
612,32	16,279	37,
51,0942	8,0917	2,41
561,2258		

4. From ,9173 subtract ,2138.   Ans. ,7035.
5. From 2,73 subtract 1,9185.   Ans. ,8115.
6. Subtract 91,713 from 407.   Ans. 315,287.
7. What is the difference between 67 and ,92 ?
                                        Ans. 66,08.

MULTIPLICATION OF DECIMALS.

RULE.* Multiply as in whole numbers ; and from the product, towards the right hand, point off as many figures for decimals, as there are decimal places in the factors. But if there be not so many figures in the product, prefix cyphers to supply the defect.

---

* The correctness of the rule may be made evident from vulgar fractions ;—Let 25,12 and 2,7 be the numbers to be multiplied. Now $25,12 = 25\frac{12}{100}$, and $2,7 = 2\frac{7}{10}$, by notation:

These reduced to improper fractions become $\frac{2512}{100}$ and $\frac{27}{10}$ respectively ; which multiplied into each other, make $\frac{67824}{1000} = 67\frac{824}{1000}$, or 67,824 giving a decimal of three figures, the number proposed in the factors.

EXAMPLES.

1.	2.
21,41	,2616
25,9	,154
19269	10464
10705	13080
4282	2616
554,519	,0402864

3. Multiply 31,72 by 65,3.   Product 2071,316.
4. Multiply ,62 by ,04.   Product ,0248.
5. Multiply 51,6 by 21.   Product 1083,6.
6. Multiply ,051 by ,0091.   Product ,0004641.

NOTE. To Multiply decimal fractions by 10, 100, 1000, &c. is only to remove the separatrix so many places towards the right hand, as there are cyphers in the multiplier.

EXAMPLES.

Multiply 64,674 by 10.   Ans. 646,74.
Multiply 3,2158 by 1000.   Ans. 3215,8.

DIVISION OF DECIMALS.

RULE. * Divide as in whole numbers; and observe the following rules for pointing off in the quotient.

1. Point off for decimals in the quotient so many figures, as the decimal places in the dividend exceed those in the divisor.

2. If the figures in the quotient are not so many as the rule requires, supply the defect by prefixing cyphers.

3. If the decimal places in the divisor be more than those in the dividend, add cyphers as decimals to the dividend, ·

---

* The reason of pointing off as many decimal places in the quotient, as those in the dividend exceed those in the divisor, will easily appear; for since the number of decimal places in the dividend is equal to those in the divisor and quotient, taken together, by the nature of multiplication; it follows that the quotient contains as many as the dividend exceeds the divisor.

until the number of decimals in the dividend be equal to those in the divisor, and the quotient will be integers until all these decimals are used. And in case of a remainder, after all the figures of the dividend are used, and more figures are wanted in he quotient, annex cyphers to the remainder, to continue the division as far as necessary.

4. The first figure of the quotient will possess the same place of integers or decimals, as that figure of the dividend which stands over the units' place of the first product.

EXAMPLES.

1. Divide 3424,6056 by 43,6.

```
    Divisor.   Dividend.    Quotient.
       43,6)3424,6056(78,546
            3052
            ____
            3726
            3488
            ____
             2380
             2180
             ____
              2005
              1744
              ____
               2616
               2616
               ____
```

2. Divide 761,2 by 2,1942.

```
        2,1942)761,2000(346,91+
               65826
               _____
               102940
                87768
               _____
                151720
                131652
                _____
                 200680
                 197478
                 _____
                  32020
                  21942
                  _____
                  10078 Remainder.
```

3. Divide 7,735 by 3,25   Ans. 2,38.
4. Divide 3877875 by ,675.   Ans. 5745000.
5. Divide 1885,78 by 7,48.   Ans. 245,42+.
6. Divide ,55736 by 48.   Ans. ,01161+.
7. Divide 7,13 by 18.   Ans. ,396+.

REDUCTION OF DECIMALS.

CASE I.

To reduce a vulgar fraction to its equivalent decimal.

RULE.* Divide the numerator by the denominator, annexing as many cyphers as are necessary; and the quotient will be the decimal required.

EXAMPLES.

1, Reduce $\frac{5}{24}$ to a decimal.

```
      24)5,0000(,20833+ Ans.
         48
         ----
         200
         192
         ----
          80
          72
          ----
           80
           72
           ----
            8 Remainder.
```

2. Required the equivalent decimal expressions for $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$.   Ans. ,25 ,5 and ,75.

3. Reduce $\frac{3}{8}$ to a decimal.   Ans. ,375.

---

* Let the vulgar fraction, whose decimal expression is required, be $\frac{7}{13}$. Now since every decimal fraction has 10, 100, 1000, &c. for its denominator; and if two fractions be equal, it will be, as the denominator of one is to its numerator, so is the denominator of the other to its nume-rator; therefore 13 : 7 :: 1000, &c : $\frac{7 \times 1000 \text{ &c.}}{13} = \frac{7000 \text{ &c.}}{13}$

=,53846+ the numerator of the decimal required; and is the same as by the rule.

4. Reduce $\frac{1}{25}$ and $\frac{33}{81}$ to decimals.

Ans. ,04 and ,407+.

5. Reduce $\frac{22}{25}$ and $\frac{1}{145}$ to decimals.

Ans. ,88 and ,00689+.

CASE II.

To reduce numbers of different denominations to their equiv-alent decimal values.

RULE.* 1. Write the given numbers perpendicularly under each other for dividends, proceeding orderly from the least to the greatest.

2. Opposite to each dividend, on the left hand, place such a number for a divisor, as will bring it to the next superior denomination, and draw a line between them.

3. Begin with the uppermost, and write the quotient of each division, as decimal parts, on the right hand of the div-idend next below it ; and so on until they are all used, and the last quotient will be the decimal sought.

EXAMPLES.

1. Reduce 15s. 9$\frac{3}{4}$d. to the decimal of a pound.

$$\begin{array}{c|l} 4 & 3, \\ 12 & 9,75 \\ 20 & 15,8125 \end{array}$$

,790625 the decimal required.

2. Reduce 19s. to the decimal of a pound.

Ans. ,95.

3. Reduce 10s. 9d. 1 qr. to the decimal of a pound.

Ans. ,5385416+.

* The reason of the rule may be explained from the first example : Thus, three farthings is $\frac{3}{4}$ of a penny, which brought to a decimal is ,75 ; consequently 9$\frac{3}{4}$d. may be expressed 9,75d. but 9,75 is $\frac{975}{100}$ of a penny $=\frac{975}{1200}$ of a shilling, which brought to a decimal is ,8125 ; and therefore 15s. 9$\frac{3}{4}$d. may be expressed 15,8125s. In like manner 15,8125s. is $\frac{158125}{10000}$ of a shilling $=\frac{158125}{200000}$ of a pound $=$, by bringing to a decimal ,790625. as by the rule.

G

4. Reduce 1 d. 2 qrs. to the decimal of a shilling.

Ans. ,125.

5. Reduce 10 oz. 18 dwt. 16 grs. to the decimal of a ℔ Troy.   Ans. ,911111+.

6. Reduce 10 oz. 14 drs. to the decimal of a hundred weight.   Ans. ,0060686+.

7. Reduce 3 rods 2½ feet 6 inches to the decimal of a mile.   ,   Ans. ,00994318+ .

8. Reduce 1 pint to the decimal of a gallon.

Ans. ,125.

9. Reduce 2 months 2 weeks 2-days to the decimal of a year.   Ans. ,197802+.

CASE III.

To find the value of any given decimal in terms of the integer.

RULE. 1. Multiply the decimal by the number of parts in the next less denomination, and cut off as many places for a remainder on the right hand, as there are places in the given decimal.

2. Multiply the remainder by the parts in the next infe-rior denomination, and cut off for a remainder as before.

3. Proceed in this manner through all the parts of the in-teger, and the several denominations, standing on the left hand, make the answer.

EXAMPLES.

1. What is the value of ,7426 of a pound?

```
            ,7426
               20
        ----------
    s. 14,8520
              12
        ----------
    d. 10,2240
              4
        ----------
          ,8960    Ans. 14s. 10¼d.
```

2. What is the value of ,384 of a shilling? Ans. 4½d.
3. What is the value of ,6725 cwt. ?

Ans. 2 qrs. 19 ℔. 5 oz+.

4. What is the value of ,61 of a tun of wine?

Ans. 2 hhds. 27 gals, 2 qts. 1 pt.

5. What is the value of ,25 of an hour ?

                  Ans. 15 minutes.

6. What is the value of ,857 of a day ?

                  Ans. 20 h. 34 m. 4 s.

7. What is the value of ,125 of a gallon ?

                  Ans. 1 pint.

· · · · · ·

RULE OF THREE.

THE RULE OF THREE teaches to find a number having
the same ratio * to a given number, which exists between
two other given numbers.

---

   * The idea of ratio results from comparing two things
of the same kind, in respect of quantity. From the simil-
itude of ratios required in the definition of the rule, results
the idea of proportion. For this reason the rule is some-
times called the rule of proportion.

   Proportion implies four terms, and these similarly pla-
ced (that is, either all the greater or all the less taken
as antecedents, and the rest as consequents) are called
proportionals. Thus the numbers 2 4, and 6 8, written
2, 4, 6, 8, or 4, 2, 8, 6, are arithmetical proportionals ; and
the numbers 2 4, and 8 16, written 2, 4, 8, 16, or 4, 2,
16, 8, are geometrical proportionals. For the method of
writing the proportionals see explanation of characters.

   Of four arithmetical proportionals, the sum of the ex-
tremes is equal to the sum of the means. Thus of 2 : 4 : :
6 ·· 8 the sum of the extremes (2+8)=the sum of the
means (4+6)=10. Therefore of three arithmetical pro-
portionals, the sum of the extremes is double the mean.

   Of four geometrical proportionals (and of this nature
are the terms of all the questions solved by the rule) the
product of the extremes is equal to the product of the
means. Thus of 2 : 4 : : 8 : 16, the product of the ex-
tremes (2×16) is equal to the product of the means (4×
8)=32. Therefore of three geometrical proportionals, the
product of the extremes is equal to the square of the mean.

   Hence it is easily seen, that either extreme of four geo-
metrical proportionals is equal to the product of the means
divided by the other extreme ; and that either mean is

RULE.* Write down the number, which is of the same kind with the answer or number required.

Consider whether the answer ought to be greater or less than this number; if greater, write the greater of the two remaining numbers on the right of it for the third, and the other on the left for the first number or term; but if it ought to be less, write the less of the two remaining numbers in the third place, and the other in the first.

Multiply the second and third terms together, divide the product by the first, and the quotient will be the answer.

NOTE 1. It is sometimes most convenient to multiply

equal to the product of the extremes divided by the other mean.

Four numbers are directly proportional, when the ratio of the first to the second is the same as that of the third to the fourth; as 2 : 4 :: 6 : 8. Four numbers are reciprocally or inversely proportional, when the first is to the second as the fourth is to the third, and *vice versa* : Thus 2 : 6 :: 9 : 3 are reciprocal proportionals ; 2 : 6 :: 3 : 9.

* The rule has its name from the number of terms given in its questions, and is founded on the principles of proportion explained in the preceding note. From these it will be seen that the magnitude of any effect varies constantly in proportion to the varying part of the cause ; thus the quantity of goods bought is in proportion to the money laid out ; the space gone over by an uniform motion is in proportion to the time, &c. The truth of the rule, as applied to ordinary inquiries, may be made evident by attending to the principles of compound multiplication and division. It is shown in multiplication of money, that the price of one, multiplied by the quantity, is the price of the whole ; and in division, that the price of the whole, divided by the quantity, is the price of one. In like manner, if the first term be any number whatever, it is plain that the product of the second and third terms will be greater than the true answer required, by as much as the price in the second term exceeds the price of one, or as the first term exceeds an unit. Consequently this product divided by the first term will give the true answer required.

and divide as in compound multiplication and division,
and sometimes it is expedient to multiply and divide ac-
cording to the rules of vulgar or decimal fractions. But
when neither of these modes is adopted, reduce each of the
compound terms to the lowest denomination mentioned in
it, and reduce the first and third to the same denomination ;
then will the answer be of the same denomination with the
second term. And the answer may afterward be brought to
any denomination required.

Note 2. *Direct* and *inverse* proportion are properly on-
ly parts of the same general rule, and are both included in
the preceding.

Two or more statings are sometimes necessary, which
may always be known from the nature of the question.

The method of proof is by inverting the question.

EXAMPLES.

1. What is the value of 2 lb. 6 oz. 19 dwts. of gold, at 3*£*.
19s. 11d. an ounce ?

```
    oz.         £.  s.  d.     lb.  oz. dwt.
    1     :   3  19  11  :  2   6   19
   20        20               12
   ──        ──               ──
   20        79               30
             12               20
                             ──
            959              619
            619

           8631
            959
           5754
          ─────

     2,0)59362,1
      ────────
       12)29681 1/2 0  Pence, or
       ──────────
       2,0)  247,3s. 5d.
       ────────
    Ans. 123£. 13s. 5+d.
```

2. If 9 lb. of tobacco cost 1 dollar 20 cents, what will 25 lb. cost ?

       lb. $. *cts.*   *lb.*
       9   :   1   20   ::   25
                          120

                    9)3000

                      —— $3,33⅓ Ans.

3. If 25 lb. tobacco cost $3,33⅓, what will 9 lb. cost ?

       lb. $, *cts.*   *lb.*
       25   :   3   33⅓   ::   9
                  9

          25)3000(120 cents, or $1,20 the answer.
           25

           50
           50

            0

4. What is the value of a firkin of butter containing 56 lb. at 10½d. per pound ?

       lb.   *d. qrs.*   *lb.*
       1   :   10   2   ::   56
                 8

$56 = 7 \times 8$ s.7   0   0
                 7

       £.2   9   0   0 the answer.

            lb.   *d.*   *lb.*
Or thus : $\frac{1}{1}$: $10\frac{1}{2} = \frac{21}{2}$ :: $\frac{56}{1}$:
        $\frac{1}{1} \times \frac{21}{2} \times \frac{56}{1} = \frac{1176}{1} = 588d. = 49s. = 2£. 9s.$

            lb.   *d.*   *lb.*
Or thus :   1   :   10,5   ::   56 :
                 56

            630
            525

         12)588,0

         2,0)4,9

         ——£,2 9s. as before.

5. If $\frac{3}{5}$ of a yard cost $\frac{7}{12}$ of a pound, what will $\frac{6}{15}$ of an ell english cost ?

First $\frac{3}{5}$ of a yard $= \frac{3}{5}$ of $\frac{4}{1}$ of $\frac{1}{5} = \frac{3 \times 4 \times 1}{5 \times 1 \times 5} = \frac{12}{25}$ of an ell.

Then $\frac{12}{25}$ ell : $\frac{7}{12}\text{\textsterling}.$ :: $\frac{6}{15}$ ell :

And $\frac{7}{12} \times \frac{6}{15} \times \frac{25}{12} = \frac{1050}{2160} = \frac{35}{72}\text{\textsterling}. = 9\text{s. } 8\frac{2}{3}\text{d. Answer.}$

6. If 6 horses eat 21 bushels of oats in a month, how many bushels will 20 horses eat in the same time ?

$$\begin{array}{ccc} Hor. & Bus. & Hor. \\ 6 & : 21 & :: 20 : \\ & 20 & \end{array}$$

$$6)420$$

$$\text{Ans. 70 bushels.}$$

7. If $\frac{3}{8}$ of a yard cost $\frac{2}{5}$ of a pound, what will $\frac{1}{4}$ of an ell english cost ?

$\frac{3}{8} =,375$ yd.

$\frac{2}{5} =,4\text{\textsterling}.$

$\frac{1}{4}$ ell $= \frac{5}{16}$ yd. $=,3125$ yd.

,375 yd. : $,4\text{\textsterling}.$ :: ,3125 yd.

,4

$$,375),12500(,333+=6\text{s. } 8\text{d. Ans.}$$
$$1125$$

$$1250$$
$$1125$$

$$1250$$
$$1125$$

$$125+\text{Remainder.}$$

8. What is the value of an cwt. of sugar at $5\frac{1}{2}$d. per lb. ?

$$\begin{array}{cccccc} lb. & d. qr. & lb. & \text{\textsterling}. & s. & d. \\ 1 & : 5 \ 2 & :: 112 & : 2 & 11 & 4 \text{ Answer.} \end{array}$$

9. How much in length of that which is $4\frac{1}{2}$ inches broad, will make a square foot ?

$$\begin{array}{cccc} Breadth. & Length. & Breadth. & Length. \\ 4\frac{1}{2} & : 12 & :: 12 & : 2 \text{ ft. 8 in. Answer.} \end{array}$$

10. Bought 6 casks of raisins each weighing 1 cwt. 1 qr. 12½ lb. what will they come to at £.2 1s. 8d. per cwt. ?

$$\begin{array}{cccccc} Cwt. & \text{\textsterling}. s. d. & Cwt. qr. lb. & \text{\textsterling}. s. d. \\ 1 & : 2 \ 1 \ 8 & :: 1 \ 1 \ 12\frac{1}{2} \times 6 & : 17 \ 0 \ 4\frac{3}{4} \text{ Ans,} \end{array}$$

11. If a man spend 2 dollars 45 cents a week, what will it amount to in a year ?

         D.   **8.** *cts.*   *D.*   **8.** *cts.*
         7 : 2 45 :: 365 : 127 75 Answer.

12. What is the value of a pipe of wine at $10\frac{1}{2}$d. per pint ?

    Pint.   *d.*   *Pipe.*   *£. s.*
     1 : 10,5 :: 1 :   44 2 Answer.

13. How many quarters of corn can I buy for 280 dollars, at $\frac{2}{3}$ of a dollar per bushel ?

                                  Ans. 52 quars. 4 bus.

14. What is the value of 2 qrs. 1 na. velvet, at 19s. $8\frac{1}{2}$d. per ell english ?   Ans. 8s. $10\frac{1}{4}$d.$\frac{7}{10}$.

15. Suppose 18 yards of broadcloth $1\frac{1}{4}$ yards wide is to be lined with shalloon that is $\frac{3}{4}$ of a yard wide ; how many yards of shalloon will be sufficient ?

                                Ans. 36 yards.

16. If 52 yards of cloth cost 156 dollars, how much will 4 yards cost ?   Ans. 12 dollars.

17. Bought 36 yards of cloth for 108 dollars, and sold the same at $3\frac{1}{2}$ dollars per yard ; how much did I gain ?

                                Ans. 18 dollars.

18. If 7 yards of ribbon cost 3s. 4d. what will 126 yards cost ?   Ans. £.3.

19. If a man earn 64 dollars in 4 months, how long must he work at the same rate to pay a debt of 300 dollars ?

                      Ans. 18 months. 3 weeks.

20. If an ounce of silver be worth 1 dollar 10 cents, what is the value of 10 silver spoons, each weighing 1 ounce 4 pennyweights ?   Ans. 13 dolls. 20 cts.

21. If $8\frac{3}{4}$ yards cost 4 dollars 20 cents, what will $13\frac{1}{2}$ yards cost ?   Ans. 6 dolls. 48 cts.

22. How long will it take 5 men to do the same work which 37 men can do in 15 days ?   Ans. 111 days.

23. What will 4 hogsheads of rum come to containing viz. $79\frac{1}{2}$, $84\frac{1}{4}$, $101\frac{1}{4}$ and 112 gallons, at 6s. 9d. per gallon ?

                          Ans. £.127 4s. 9d.

24. If 1 and 2 make 7, what will 3 and 6 make ?

                                  Ans. 21.

25. If a chest of hyson tea weighing 79 lb. neat, cost £.32 11s. 9d. what is it per pound ?

                              Ans. 8s. 3d.

6. B owes £.2119 17s. 6d. and he is worth but £.1324
5¼d. ; if he delivers this to his creditors, how much
hey receive on a pound ?   Ans. 12s. 6d.

7. A merchant failing in trade owes in all 29475 dol-
lars, and delivers up his whole property worth 21894 dol-
lars 3 cents ; how much per cent does he pay ; and what is
B's loss to whom he owed 325 dollars ?

Ans. He pays $74 28 cts. per cent.
And B loses $83 59 cents.

28. If a staff, 4 feet 8 inches in length, cast a shadow 6
feet ; how high is that steeple whose shadow is 153 feet ?
Ans. 119 feet.

29. Bought 270 quintals codfish for 780 dollars ; freight
37 dollars 70 cents ; wharfage, truckage and other expences
30 dollars 60 cents ; at what must I sell them per quintal
so as to gain 143 dollars on the whole ?

Ans. $3 67 cts. 1 mill+.

30. If ⅗ of a farm cost 1081 dollars, what is the whole
worth ?   Ans. $1801 66 cts. 6 mills+.

31. If a man spend 46 cents a day, what will it amount
to in a year ?   Ans. $167,90.

32. Lent a friend 292 dollars for six months ; some-
time afterward he lent me 403 dollars ; how long must I
keep it to balance the favor ?

Ans. 4 m. 1 w. 2 d+.

33. If 100 dollars gain six dollars interest in one year,
how much will 480 dollars gain in the same time ?

Ans. $28,80.

34. If 480 dollars gain 28 dollars 80 cents in one year,
how much will it gain in 87 days ?

Ans. 6 dolls. 86 cts. 4 mills.

35. How much land, at 2 dollars 50 cents per acre,
must be given in exchange for 360 acres, at 3 dollars 75
cents per acre ?   Ans. 540 acres.

36. Bought a silver cup weighing 9 ounces 4 penny-
weights 16 grains, for £.3 2s. 3d. 3 qrs ⅔ ; what was it per
ounce ?   Ans. 6s. 9d.

37. There is a cistern which has four cocks ; the first
will empty it in 10 minutes, the second in 20 minutes, the
third in 40 minutes, the fourth in 80 minutes ; in what
time will all four running together empty it ?

Ans. 5 min. 20 sec.

38. A hired two men, B and C, to cut wood for 50 cents per cord ; B could cut a cord in 4 hours, C in 6 hours ; how long would it take both to cut 1 cord ?

                                    Ans. 2 hours 24 minutes.

39. If, when wheat is 6s. 3d. per bushel, the penny loaf weigh 9 ounces, what ought it to weigh when wheat is 8s. 2½d. per bushel ?   Ans. 6 oz. 13 drs.

40. When a man's yearly income is 949 dollars, how much is it per day ?   Ans. 2 dolls. 60 cts.

41. What is the commission on 1525 dollars, at 4½ dolls. per cent ?   Ans. 68 dolls. 62 cts. 5 mills.

42. What will 374 feet of boards come to at 1½ cent per foot ?   Ans. $5 61 cts.

43. What will 39 thousand 6 hundred and 30 casts of staves come to at 15 dolls. 50 cts. per thousand ?

                            Ans. $614 96 cts. 2½ mills.

Note. 3 staves make 1 cast ; 40 casts 1 hundred ; 10 hundred 1 thousand.

44. If the inventory of a town be 358400 dollars, upon which there is assessed a tax of 850 dollars, what will it be on a dollar ; and what will B's tax be whose estate in that town is valued at 1792 dollars ?

                    Ans. 2 mills $\dfrac{37+}{100}$ on a dollar.

                    And B's tax will be 4 dolls. 25 cts.

45. What will the charter of a ship of 306 tons amount to, from May 28 to October 10th following, at 2 dollars per ton, per month of 30 days ?

                            Ans. 2774 dolls. 40 cts.

Note. The days of receiving and discharging are both included.

46. If 4½ hundred weight may be carried 36 miles for 35s. how many pounds can I have carried 20 miles for the same money ?   Ans. 907⅕ lb.

47. If, when the days are 13⅝ hours long, a traveller performs his journey in 35¼ days ; in how many days will he perform the same journey when the days are 11 9/16 hours long ?   Ans. 40 615/952 days.

48. What quantity of water must I add to a pipe of mountain wine valued at 38£. to reduce the first cost to 4s. 6d. per gallon ?   Ans. 20⅔ gallons.

49. A and B depart from the same place, and travel the same road ; but A goes 6 days before B, at the rate of 21 miles a day ; B follows at the rate of 28 miles a day ; in what time and distance will he overtake A ?

$$\text{Ans.} \begin{cases} 18 \text{ days.} \\ 504 \text{ miles.} \end{cases}$$

50. If the minute hand of a clock move round 12 times while the hour hand move once round ; what is the period of its passing the hour hand ?

Ans. 1 h. 5 min. $27\frac{3}{11}$ sec.

51. It is now 6 o'clock, the hands of my watch are directly opposite ; in what time will the minute hand, moving twelve times as fast as the hour hand, be in the same direction ?   Ans. 32 min. $43\frac{7}{11}$ sec.

52. A person being asked the time of day, answered, it is between 4 and 5 ; but a more particular answer being required, he said, that the hour and minute hands were then exactly together ; what was the time ?

Ans. $21\frac{9}{11}$ min. past 4.

53. How many yards of cloth 3 qrs. wide, will be equal in measure to 30 yards 5 qrs. wide ?   Ans. 50 yards.

. . . . . .

TARE AND TRET.

TARE AND TRET are practical rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

Tare is an allowance made to the buyer for the weight of the box, barrel, or bag, &c. which contains the goods bought, and is either at so much per box, &c. at so much per cwt. or at so much in the gross weight.

Tret is an allowance of 4 lb. in every 104 lb. for waste, dust, &c.

Cloff is an allowance of 2 lb. upon every 3 cwt.

Gross weight is the whole weight of any sort of goods, together with the box, barrel, or bag, &c. that contains them.

Suttle is the weight when part of the allowance is deducted from the gross.

Neat weight is what remains after all allowances are made.

Case I.

When the tare is a certain weight per box, barrel, or bag, &c.

RULE. Multiply the number of boxes, or barrels, &c. by the tare, and subtract the product from the gross, and the remainder is the neat weight required.

EXAMPLES.

1. In 10 casks of allum, each weighing 3 cwt. 2 qrs. 12 lb. gross, tare 15 lb. per cask, how much neat?

```
              Cwt. qr.  lb.
               3   2    12
                        10
              _____
              36   0    8  Gross.
15×10=150 lb= 1    1   10  Tare.
              _____
              34   2   26  The answer.
```

2. In 241 barrels of figs, each 3 qrs. 19 lb. gross, tare 10 lb. per barrel, how many pounds neat? Ans. 22413 lb.

3. What is the neat weight of 21 hogsheads of tobacco, each 5 cwt. 2 qrs. 17 lb. gross, tare 100 lb. per hogshead? Ans. 99 cwt. 3 qrs. 21 lb.

4. What is the neat weight of 4 chests of hyson tea, weighing gross 96 lb. 97 lb. 101 lb. and 103 lb. tare 20 lb. per chest? Ans. 317 lb.

Case II.

When the tare is a certain weight per cwt.

RULE. Divide the gross weight by the aliquot * parts of an cwt. contained in the tare, and subtract the quotient from the gross, and the remainder is the neat weight.

_____

* An aliquot part of any number is such a part of it as, being taken a certain number of times, exactly makes that number.

TABLE OF ALIQUOT PARTS.

Parts of an cwt.		Parts of ½ cwt.		Parts of ¼ cwt.	
2 qrs. is	$\frac{1}{2}$	28 lb. is	$\frac{1}{2}$	14 lb. is	$\frac{1}{2}$
1	$\frac{1}{4}$	14	$\frac{1}{4}$	7	$\frac{1}{4}$
16 lb.	$\frac{1}{7}$	8	$\frac{1}{7}$	4	$\frac{1}{7}$
14	$\frac{1}{8}$	7	$\frac{1}{8}$	3½	$\frac{1}{8}$
8	$\frac{1}{14}$	4	$\frac{1}{14}$	2	$\frac{1}{14}$
7	$\frac{1}{16}$	3½	$\frac{1}{16}$	1	$\frac{1}{28}$
4	$\frac{1}{28}$	2	$\frac{1}{28}$		
2	$\frac{1}{56}$				

EXAMPLES.

1. Gross 372 cwt. 3 qrs. 17 lb. tare 16 lb. per cwt. how much neat ?

Cwt. qrs. lb.

16 lb. is ⅐ 372 3   17
               53 1   2⅐ tare subtracted.

              319 2 14⅘ the answer.

2. What is the neat weight of 7 barrels of potash, each weighing 402 lb. gross, tare 10 lb. per cwt. ?

Ans. 2562 lb. 12 oz.

3. In 129 cwt. 3 qrs. 16 lb. gross, tare 14 lb. per cwt. what is the neat weight ?

Ans. 113 cwt. 2 qrs, 17¼ lb.

4. In 25 barrels of figs, each 2 cwt. 1 qr. gross, tare 16 lb. per cwt. how much neat ?

Ans. 48 cwt. 0 qrs. 24 lb.

. . . . . .

DOUBLE RULE OF THREE.

THE DOUBLE RULE OF THREE teaches to solve such ques-tions as require two or more statings in the rule of three. In these questions there is always given an odd number of terms, as five, seven, or nine, &c. These are distinguish-ed into *terms of supposition*, and *terms of demand*, the num-ber of the former always exceeding that of the latter by one, which is the same kind with the term or answer sought.

RULE.* Write the term of supposition, which is of the same kind with the answer, for the middle term.

Take one of the other terms of supposition, and one of the demanding terms of the same kind with it ; then place one of them for a first term, and the other for a third, ac-cording to the directions given in the rule of three. Do. the same with another term of supposition and its corres-ponding demanding term ; and so on if there be more terms

* The reason of this rule for stating, and of the meth-ods of operation, is shown from the nature of simple pro-portion.

H

of each kind, writing the terms under each other, which
fall on the same side of the middle term.

Multiply together all the terms in the first place, and
also all the terms in the third place. Then multiply the
latter product by the middle term, and divide the result by
the former product; and the quotient will be the answer
required.

Note. The first and third terms of each line, if of dif-
ferent denominations, must be reduced to the same denom-
ination.

Examples.

1. How many men can complete a trench of 135 yards
long in eight days, provided 16 men can dig 54 yards in 6
days?

$$\left. \begin{array}{c} 54\ \text{yards} \\ 8\ \text{days} \end{array} \right\} \ \ : \ \ 16\ \text{men}\ : \ \ \left\{ \begin{array}{c} 135\ \text{yards} \\ 6\ \text{days} \end{array} \right\} :$$

$$\begin{array}{c} 432 \end{array} \qquad\qquad \begin{array}{r} 810 \\ 16 \\ \hline 4860 \\ 810 \\ \hline \end{array}$$

$$432)12960(30\ \text{Men, ans.}$$
$$\underline{1296}$$
$$0$$

2. If 100£. in one year gain 6£. interest, what will be
the interest of 750£. for 7 years?

$$\left. \begin{array}{c} 1\ \text{year} \\ 100\pounds. \end{array} \right\} \ \ : \ \ 6\pounds. \ \ :: \ \ \left\{ \begin{array}{c} 7\ \text{years} \\ 750\pounds. \end{array} \right\} :$$

$$\begin{array}{c} \overline{100} \end{array} \qquad\qquad \begin{array}{r} 7 \\ \hline 5250 \\ 6 \\ \hline \end{array}$$

$$1,00)315,00$$

Answer 315£.

3. A farmer sells 204 dollars worth of grain in 5 years,
when it sold at 60 cents per bushel; what is it per bushel

when he sells 1000 dollars worth in 18 years, if he sell the same quantity yearly ?   Ans. 81 cts. 6$\frac{9}{10}$+mills.

4. If 7 men can reap 84 acres of wheat in 24 days, how many men can reap 100 acres in 10 days ?

Ans. 20 men.

5. If 6 men build a wall 20 feet long, 6 feet high and 4 feet thick, in 16 days ; in what time will 24 men build one 200 feet long, 8 feet high and 6 feet thick ?

Ans. 80 days.

$$\left.\begin{array}{l}\text{24 men}\\\text{20 feet long}\\\text{6 feet high}\\\text{4 feet thick}\end{array}\right\} : \text{16 days} :: \left\{\begin{array}{l}\text{6, men}\\\text{200 feet long}\\\text{8 feet high}\\\text{6 feet thick}\end{array}\right\} : \text{80 days.}$$

6. An usurer put out 75 dollars at interest, and at the end of 8 months he received for principal and interest, 79 dollars ; I demand what rate per cent. he received interest ?

Ans. 8 per cent.

7. If the carriage of 13 cwt. 1 qr. for 72 miles be £.2 10s. 6d. what will be the carriage of 7 cwt. 3 qrs. for 112 miles ?

Ans. £.2 5s. 11d. 1$\frac{77}{159}$qr.

8. If a family of 9 persons spend 450 dollars in 5 months, how much would be sufficient to maintain them 8 months, if 5 more were added to the family ?

Ans. 1120 dollars.

9. What is the interest of 654 dollars for 164 days, at 6 per cent. per annum ?   Ans. 17 dolls. 63 cts. 1 m.

10. If 248 men in 5 days of 11 hours each, dig a trench 230 yards long, 3 yards wide and 2 deep ; in how many days of 9 hours long, will 24 men dig a trench 420 yards long, 5 wide and 3 deep ?   Ans. 288$\frac{59}{307}$ days.

11. If 10 lb. at Boston make 9 lb. at Amsterdam ; 90 lb. at Amsterdam 112 lb. at Thoulouse ; how many pounds at Thoulouse are equal to 50 lb. at Boston ?

Ans. 56 lb.

12. If 25 lb. at Boston be 22 lb. at Nuremburgh ; 88 lb. at Nuremburg 92 lb. at Hamburg ; 46 lb. at Hamburgh 49 lb. at Lyons ; how many pounds at Boston are equal to 98 lb. at Lyons ?   Ans. 100 lb.

B A R T E R.

BARTER is the exchanging of one commodity for another, and directs traders so to proportion their goods, that neither party may sustain loss.

RULE.* Find the value of that commodity whose quantity is given ; then find what quantity of the other, at the rate proposed, you may have for the same money, and it gives the answer required.

EXAMPLES.

1. How many dozen of candles at 3s. 6d. per dozen must be given in barter for 4 cwt. 2 qrs. of tallow, at 46 shillings per cwt. ?

```
            qrs.    s.        cwt. qrs.
            4  :  46    : :    4   2   :
                  18           4
                  ——          ——
                  368          18
                  46
                  ——
               4)828
               ————
             2,0)20,7
               ————£.10 7s,
       s.  d.    doz.        £.  s.
       3   6  :   1    : :   10  7.
      12                     20
      ——                    ———
      42                    207
                            12
                           ———
                   42)2484(59 doz. 1+ Ans.
                      210
                      ———
                      384
                      378
                      ———
                        6
                       12
                       ——
                   42)72(1
                      42
                      ——
                      80
```

—————————————————————————————
* This rule is only an application of the rule of three.

2. A buys of B 4 hogsheads of rum containing 410 gallons, at 1, dollar 17 cents per gallon ; and 253 lb. of coffee at 21 cents per pound : In part of which he pays him 21 dollars in cash, and the balance in boards at 8 dollars per thousand ; how many feet of boards does the balance require ?

<div style="text-align: right">Ans. 63978¾ feet.</div>

3. Bought a sloop of 70 tons at 16 dollars per ton ; paid in cash 500 dollars, 350 gallons of molasses at 64 cts. per gallon, and the balance in New England rum at 74 cts. per gallon ; how many gallons did it amount to ?

<div style="text-align: right">Ans. 535$\frac{5}{37}$.</div>

4. A barters with B 150 bushels of wheat at 5s. 9d. per bushel, for 65 bushels of corn at 2s. 10d. per bushel, and the balance in oats at 2s. 1d. per bushel ; what quantity of oats must A receive ?

<div style="text-align: right">Ans. 325$\frac{3}{5}$ bushels.</div>

5. How much wine at 1 dollar 28 cents per gallon, must I receive in barter for 26 cwt. 2 qrs. 14 lb. of raisins, at 9 dollars 44 cents 4 mills per cwt. ?

<div style="text-align: right">Ans. 196 gals. 1 qt. 1 pt. 2 gills.</div>

6. A delivers B 3 hogsheads of brandy at 6s. 8d. per gallon, for 126 yards of cloth ; what was the cloth per yard ?

<div style="text-align: right">Ans. 1 dollar 66$\frac{2}{3}$ cts.</div>

7. A has a quantity of pepper, weight neat 1600 lb. at 1s. 5d. per lb. which he barters with B for two sorts of goods, the one at 5d. the other at 8d. per pound, and to have $\frac{1}{3}$ in money, and of each sort of goods an equal quantity ; how many lb. of each must he receive, and how much in money ?

<div style="text-align: right">Ans. 1394$\frac{34}{39}$ lb. of each, and £.37 15s. 6$\frac{3}{4}$d.</div>

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LOSS AND GAIN.

Loss and Gain is a rule that discovers what is gained or lost in buying or selling goods ; and instructs merchants and traders to raise or lower the price of their goods, so as to gain or lose a certain sum per cent.

Questions in this rule are performed by the rule of three.

<div style="text-align: center">H 2</div>

EXAMPLES.

1. Bought 30 hogsheads of molasses, at 600 dollars ; paid in duties 20 dollars 66 cents, for freight 40 dollars 78 cents, for storage 6 dollars 5 cents, and for insurance 30 dollars 84 cents : If I sell it at 26 dollars per hogshead, how much shall I gain per cent. ?

```
        S.  cts.
       600                    $26
        20  66                 30 Hhds.
        40  78                ─────
         6  05                $780  00 Sold for
        30  84                 698  33 Cost.
       ───────               ───────────
       698  33                  81  67 Gain.
        S.  cts.     S.  cts.        S.
       698  33   :   81  67    ::   100
                         100
                     ─────       S.  cts.
              69833)816700(11  69+ Ans.
                    69833
                    ──────
                    118370
                     69833
                    ──────
                    4843700
                     418998
                    ──────
                     653720
                     628497
                    ──────
                      25223
```

2. At 3s. 6d. profit on the pound, how much per cent. ?
                                    Ans. 17£. 10s.

3. If 1 lb. of coffee cost 28 cts. and it sold for 31 cents, what is the profit on 293 lb. neat?   Ans. 8 dolls. 79 cts.

4. If a gallon of wine cost 6s. 8d. and is sold for 7s. 2d. what is the gain per cent. ?   Ans. 7½ per cent.

5. Sold a repeating watch for 175 dollars, upon which I lost 17 per cent. whereas I ought to have gained 20 per cent, how much was it sold for under its just value ?
                                    Ans. 78 dolls. 1 et +½

6. If I buy broadcloth for 13s. 5d. per yard, how must I sell it to gain at the rate of 25 per cent.?

$$\begin{array}{ccc} \pounds. & \pounds. & s. \ d. \\ 100 \ : \ & 125 \ :: \ & 13 \ 5 \\ & 161 \ \ & 12 \end{array}$$

$$\begin{array}{cc} 125 & 161 \\ 750 & \\ 125 & \end{array}$$

$$1,00)201,25 \qquad \text{Or thus.}$$

$$12)201 \qquad\qquad 4)13 \ 5$$
$$\qquad\qquad\qquad 3 \ 4\tfrac{1}{2}$$

16s. 9d. $\frac{24}{100}$ the ans.   16s. 9$\frac{1}{4}$d.

7. Bought rum for 90 cents per gallon; at what rate must it be sold to gain 20 per cent.?   Ans. 108 cts.

8. Bought 115 gallons of rum at 1 dollar 10 cents per gallon; how many gallons of water must be put in so as to gain 5 dollars by selling it at 1 dollar per gallon?
Ans. 16$\frac{1}{4}$ gallons.

. . . .

FELLOWSHIP.

FELLOWSHIP is a rule by which merchants &c. trading in company with a joint stock, determine each person's particular share of the gain or loss in proportion to his share in the joint stock.

By this rule a bankrupt's estate may be divided among his creditors; as also legacies adjusted when there is a deficiency of assets or effects.

SINGLE FELLOWSHIP.

Single Fellowship is when different stocks are employed for any certain equal time.

RULE.* As the whole stock is to the whole gain or loss,

* That the gain or loss in this rule is evidently in proportion to their stocks, may be shown from the nature of the rule of three.

so is each man's particular stock to his particular share of the gain or loss.

PROOF. Add all the shares together, and the sum will be equal to the gain or loss, when the work is right.

EXAMPLES.

1. A and B gained by trade 182£. A put into stock 300£, and B 400£. what is each person's share of the profit?

$$
\begin{array}{ccccc}
£. & £. & £. & £. & £. \\
300+400=700 & : & 182 & :: & 300 \; ; \\
& & 300 & & \\
\end{array}
$$

7,00)546,00

78£.

$$
\begin{array}{ccccc}
£. & & £. & & £. \\
700 & : & 182 & :: & 400 \; ; \\
& & 400 & & \\
\end{array}
$$

7,00)728,00

104£.

Ans. $\begin{cases} £.78 \text{ A's share.} \\ £.104 \text{ B's share.} \end{cases}$

£.182 proof.

2. A man dying bequeathed his estate to his three sons in the following manner, viz. to the eldest he gave 1840 dollars, to the second 1550 dollars, and to the third 960 dollars; but it was found his whole estate was no more than 1840 dollars; what is each one's proportion?

Ans. $\begin{cases} \$778,29\frac{4}{5} \text{ the first.} \\ 655,63\frac{2}{5} \text{ the second} \\ 406,06\frac{2}{5} \text{ the third.} \end{cases}$

3. A and B companied; A put in 450 dollars and received $\frac{3}{5}$ of the gain; what did B put in?

Ans. 300 dollars.

4. Three merchants freight a ship with wine; A loaded 110 tuns, B 97 tuns, and C 133 tuns. In a storm the sea-

men were obliged to throw 85 tons overboard ; how much must each sustain of the loss ?

Ans. A 27¼, B 24½, C 33¼ tuns.

5. Three men, A B and C contract to build the hull of a vessel for 625 dollars.; A works 100 days and his work is estimated at 1 dollar 80 cents per day ; B works 101½ days estimated at 1 dollar 60 cents per day, and C works 98 days at 1 dollar 50 cents per day ; how much is each man's proportion according to his work ?

day. $.cts. days. day. $.cts. days. day. $.cts. days.
1 : 1 80 :: 100 : 1 : 1 60 :: 101½ : 1 : 1 50 :: 98
　　100　　　　　　　　1 01½　　　　　　　　150

180,00 A's work.　　40　　　　　　　4900
162,00 B's do.　　160　　　　　　　98
147,00 C's do.　　160　　　　　　———
　　　　　　　　　———　　　　　　・ 147,00
489　　　　　162,00

　　　　　$.　　$.　　$.　　$. cts.
　　　　489 : 625 :: 162 : 207,05⅔
　　　　489 : 625 :: 147 : 187,88⅔

　　$.　　　$.　　　$.
489 : 625 :: 180 $
　　　180

　　50000
　　625
　　———　　　$. cts.
489)112500(230,06⅔
　　978

　　1470
　　1467
　　———
　　　3000
　　　2934
　　　———
　　　66+　　⎧ $230,06+ A's share.
　　Ans.⎨ $207,05⅓+ B's do.
　　　　　⎩ $187,88⅓+ C's do.

　　　　$625,00 Proof.

6. A ship worth 3600 dollars being entirely lost, of which ½ belonged to A, ¼ to B, and the rest to C ; what loss will each sustain ?

Ans. A $450.　B $900.　C $2250.

7. A and B gained 1260 dollars, of which A is to have ten per cent. more than B ; what is the share of each ?

Ans. A $660.　B $600.

8. Three merchants made a joint stock—A put in £.565 6s. 8d. B £.478 5s. 4d. and C a certain sum ; they gained £.373 9s. 11d. of which C took £.112 11s. 11d. for his part ; what is A and B's part of the gain, and how much did C put in ?

Ans. $\begin{cases} \text{A's gain £.141 6s. 8d.} \\ \text{B's do. £.119 11s. 4d.} \\ \text{C put in £.450 7s. 8d.} \end{cases}$

DOUBLE FELLOWSHIP.

Double Fellowship is when the stocks are employed for different times.

RULE.* Multiply each man's stock by the time of its continuance ; then say, as the sum of all the products is to the whole gain or loss, so is each man's particular product to his particular share of the gain or loss.

EXAMPLES.

1. A and B hold a piece of ground in common, for which they pay £.36—A put in 23 oxen 54 days, B 21 oxen for 70 days ; what part of the rent must each man pay ?

```
23×54=1242
21×70=1470
                                      £.  s.  d.
    £.                    { 1242 :  16  9  8¾ A's.
  2712 : 36    : :        { 1470 :  19 10  3¼ B's.
                                   ─────────────
                             £.36 Proof.
```

* When the times are equal, the shares of the gain or loss are evidently as the stocks, as in single fellowship ; and when the stocks are equal the shares are as the times ; but when neither are equal, the shares must be as their products.

2. Two merchants enter into partnership for 16 months. A put in at first 1200 dollars, and at the end of 9 months 200 dollars more ; B put in at first 1500 dollars, and at the expiration of 6 months took out 500 dollars—with this stock they gained 772 dollars 20 cents ; what is each man's part of it?

Ans. A's $401 70 cents—B's $370 50 cents.

3. A B and C enter into partnership ; A put in 85 dollars for 8 months, B put in 60 dollars for 10 months, and C 120 dollars for 3 months ; by misfortune they lost 41 dollars ; what part of the loss must each man sustain ?

Ans. A's part $17. B's $15. C's $9.

. . . .

SIMPLE INTEREST.

SIMPLE INTEREST is a compensation for the use of money according to a certain rate per cent. agreed on for the principal only.

The legal interest in most of the United States is 6 per cent. per annum.

Principal is the money for which the compensation is made.

Rate is the sum per cent. agreed on.

Amount is the principal and interest added together.

RULE.* Multiply the principal by the rate, and divide the product by 100, and the quotient is the interest for one year. Multiply the interest for one year by the given number of years, and the product is the interest for that time : For any parts of a year, as months, days, &c. divide the interest for one year by the aliquot parts of a year or month ; or the interest may be found by a statement in the rule of three.

NOTE. In federal money the quotient after the division by 100 gives the answer in the same name with the lowest denomination in the principal.

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* Simple Interest is only an application of the rule of three.

TABLE OF ALIQUOT PARTS.

Parts of a Year.		Parts of a Month.	
6 Months is	$\frac{1}{2}$	3 Days is	$\frac{1}{10}$
4	$\frac{1}{3}$	5	$\frac{1}{6}$
3	$\frac{1}{4}$	6	$\frac{1}{5}$
2	$\frac{1}{6}$	10	$\frac{1}{3}$
$1\frac{1}{2}$	$\frac{1}{8}$	15	$\frac{1}{2}$
1	$\frac{1}{12}$		

EXAMPLES.

1. What is the interest of 639$. for one year, at 6 per cent. ?

$$639$$
$$6$$
—
$$\pounds.38,34$$
$$20$$
—
$$s.6,80$$
$$12$$
—
$$d. 9,60$$
$$4$$
—
$$qr.2,40$$
—— Ans. 38$. 6s. 9$\frac{1}{2}$d+.

2. What is the interest of 372 dollars for one year 5 months and 5 days, at 6$\frac{1}{2}$ per cent. ?

$$372$$
$$6\frac{1}{2}$$
—
$$2232$$
$$186$$
———
$24,18 Interest for one year.

4 months, $\frac{1}{3}$	8,06	do.	four months.
1 month, $\frac{1}{12}$	2,01$\frac{1}{2}$	do.	one do.
5 days, $\frac{1}{6}$ of 1 mo.	33$\frac{1}{2}$	do.	five days.

$34,59 Answer

3. What is the amount of £.49 6s. 4½d. for one year, at 6 per cent. ?

```
            £.  s.   d.
           49   6   4½
                     6
           ─────────────
         2,95  18   3
           20
           ─────────────
         19,18          £.49  6  4½  Principal.
           12              2  19  2   Interest.
           ─────────────   ───────────
         2,19      Ans. £.52  5  6½  Amount.
```

4. What is the interest of 1600 dollars for one year and three months, at 6 per cent. ?

Ans. $120.

5. What is the interest of £.71 7s. 6½d. for 2 years, at 6 per cent. ?

Ans. £.8 11s. 3½d.

6. What is the interest of 67 dollars 62 cents for 3 years and 2 months, at 6 per cent. ?

Ans. $12 84 cts. 7 mills.

7. How much is the interest of 325 dollars for 3 years, at 6 per cent. ?

Ans. $58 50 cts.

8. How much is the interest of 66 cents 4 mills for 1 year and 7 months, at 6 per cent. ?

Ans. 6 cts. 3 mills.

9. What is the interest of 48 dollars 25 cents 5 mills for 5 years, at 5 per cent. ?

Ans. $12 6 cts. 3 mills.

10. What is the interest of 48 dollars for 3 years, at 9 per cent. ?

Ans. $12 96 cts.

11. What is the interest of £.5 16s. 3d. for one year and 6 months, at 6 per cent. ?

Ans. 10s. 5½d.

12. What is the interest of 9672 dollars for 2 years 7 months and 4 days, at 8 per cent. ?

Ans. $2007 47 cts, 7 mills.

I

*To find the interest of any sum at 6 per cent. per annum,
for any number of months.*

RULE.* Multiply the principal by half the number of
months, and that product divided by 100 will be the interest for the given time.

EXAMPLES.

1. What is the interest of 64 dollars 50 cents for 8
months, at 6 per cent. ?

$$64,50$$
$$8 \div 2 = 4 \qquad 4$$
$$\overline{}$$
$$1,00)2,58,00 \text{ Ans. 2 dolls. 58 cents.}$$

2. How much is the interest of 36 dollars 84 cents for 5
months, at 6 per cent. ?

                          Ans. 92 cts. 1 mill.

3. How much is the interest of 750 dollars for 15 months,
at 6 per cent. ?

                          Ans. $56 25 cts.

4. What is the interest of £.24 15s. 4½d. for 10 months,
at 6 per cent. ?

                          Ans. £.1 4s. 9¼d.

5. What is the interest of 468 dollars for one month, at
6 per cent. ?

                          Ans. $2 34 cts.

*To find the interest of any sum for any number of days, when
the rate is 6 per cent.*

RULE. Multiply the principal by the number of days,
and divide the product by 6083 ; the quotient will be the
interest required.

---

* THE REASON OF THE RULE. When the time is months,
multiplying by the rate for the time gives the answer.

This rate is found by multiplying the time by the given
rate per cent. for a year, and dividing the product by 12
the quotient is the rate required, and is always equal to
half the months, when the yearly rate is 6 per cent.

EXAMPLES.

1. What is the interest of 376 dollars 20 cents for 80 days, at 6 per cent.? 376,20

```
                        80
                    ───────── $. cts. m.
        6083)30096,00(4 94 7+ Answer.
             24332

             57640
             54747

             28930
             24332

             45980
             42581

             3399+
```

2. What is the interest of £.749 10s. 6d. for 12 days, at 6 per cent.?

```
                £.    s.   d.
               749   10   6
                          12

    6083)8994    6    0(1£. 9s. 6¾d. Answer,
         6083

         2911.
           20

    6083)58226(9s.
         54747

          3479
            12

    6083)41748(6d.
         36498

          5250.
             4

    6083)21000(3qr.
         18249

          2751
```

COMPOUND INTEREST.

COMPOUND INTEREST is what arises from the interest being added to the principal, and becoming a part of the principal, at the end of each stated time of payment.

RULE. Find the simple interest of the given sum for one year, or the time of the first payment ; add it to the principal, and find the interest of the amount for the next year or payment, and so on for the number of payments required. Subtract the principal from the last amount, and the remainder will be the compound interest.

EXAMPLES.

1. What is the compound interest of 406 dollars for 3 years, at 6 per cent. per annum ?

$$406 \text{ principal for the 1st year.}$$
$$6$$

$$\begin{array}{l} .24,36 \text{ interest of do.} \\ 406, \quad \text{principal for the 1st year.} \end{array} \Big\} \text{ Add.}$$

$$430,36 \text{ principal for the 2d year.}$$
$$6$$

$$\begin{array}{l} 25,82,1,6 \text{ interest of do.} \\ 430,36 \quad \text{principal for the 2d year.} \end{array} \Big\} \text{ Add.}$$

$$456,18,1 \quad \text{principal for the 3d year.}$$
$$6$$

$$\begin{array}{l} 27,37,0,86 \quad \text{interest for do.} \\ 456,18,1 \quad \text{principal for the 3d year.} \end{array} \Big\} \text{ Add.}$$

$$483,55,1 \quad \text{amount for 3 years.}$$
$$406, \quad \text{principal for the 1st year, subtracted.}$$

Ans. $77,55,1 compound interest.

2. How much is the compound interest of 2535 dollars for four years, at 6 per cent. per annum ?

Ans. 665 dolls. 36 cts.

3. What is the compound interest of 1000 dollars for 5 years, at 6 per cent. ?

Ans. 338 dolls. 22 cts. 4 mills.

4. What is the compound interest of £.128 17s. 6d. for 6 years, at 6 per cent. ?   Ans. £.53 18s. 8¾d.

5. How much will 680 dollars amount to in 4 years, at 6 per cent. compound interest ?

```
             $. cts.
          680,
           40,80   first year's interest.
           ------
          720,80   amount of the first year.
           43,24,8 second year's interest.
           ------
          764,04,8 amount of the second year.
           45,84,2 third year's interest.
           ------
          809,89,0 amount of the third year.
           48,59,3 fourth year's interest.
           ------
 Ans. $858,48,3 amount of the fourth year.
           ------
```

. . . . . .

COMMISSION.*

COMMISSION AND BROKERAGE are compensations to factors and brokers for their respective services.

EXAMPLES.

1. What is the commission on 4760 dollars, at 2½ per cent.?

```
          2) 4760
               2½
             -----
           . 9520
             2380
             -----
           119,00 Ans. 119 dollars.
```

* The method of working questions in this and the following rules of Insurance, &c. is the same as in simple interest.

2. What is the commission on £.526 11s. 5d. at 3½ per cent. ?   Ans. £.18 8s. 7d.

3. What is the brokerage on 926 dollars 50 cents, at 1½ per cent. ?   Ans. 13 dolls. 89 cts. 7 mills.

4. What is the commission on 1298 dollars 53 cents, at ¾ per cent. ?   Ans. 9 dolls. 73 cts. 8 mills.

5. Required the neat proceeds of certain goods amounting to 2176 dollars, deducting a commission of ⅞ per cent. ?
            Ans. 2156 dolls. 96 cts.

6. A factor receives 3690 dollars to lay out in potash, reserving from it his commission of 2¼ per cent. on the purchase ; the potash being 190 dollars per ton, how much did he purchase ?
            Ans. 18 tons, 18 cwt. 3 qrs. 22²⁄₁₉ lb.

. . . .

INSURANCE.

INSURANCE is an exemption from hazard, by paying a certain sum on condition of being indemnified for loss or damage of ships, houses, merchandize, &c. which may happen from storms, fires, &c.

EXAMPLES.

1. What is the premium of insuring 8250 dollars, at 6 per cent. ?

$$8250$$
$$6$$

495,00 Ans. 495 dollars.

2. What is the premium of insuring $1650 at 15½ per cent. ?
            Ans. 255 dolls. 75 cts.

3. What sum must be received for a policy of 1658 dollars, deducting a premium of 23 per cent. for insurance ?
            Ans. 1276 dolls. 66 cts.

4. What is the premium for the insurance of 4000 dollars, at 7⅝ per cent. ?
            Ans. 305 dollars.

5. What sum must be insured upon to cover 1800 dollars, when the premium is 10 per cent. ?

             100 Policy.
    Deduct 10 Premium.
           _____

           90 Sum covered.
             If $90 : $100 :: $1800 : $2000
                                   Ans. 2000 dollars.

. . . .

DISCOUNT.

Discount is an allowance made for the payment of any sum of money before it becomes due ; and is the difference between that sum due some time hence, and its present worth. The *present worth* of any sum, due some time hence, is such a sum, as, if put to interest, would in that time, and at the rate per cent. for which the discount is to be made, amount to the sum or debt then due.

RULE.* As the amount of 100 dollars for the given rate and time is to the interest of 100 dollars for that time, so is the given sum or debt to the discount required.

* That an allowance ought to be made for paying money before it becomes due, which is supposed to bear no interest until after it is due, is very reasonable ; and this allowance ought to be such a sum, as being put to interest until the debt becomes due, would amount to the interest of the debt for the same time.

The truth of the rule for working is evident from the nature of simple interest ; for since the debt may be considered as the amount of some principal (called here the present worth) at a certain rate per cent. and for the given time, that amount must be in the same proportion, either to its principal or interest, as the amount of any other sum at the same rate and for the same time, is to its principal or interest.

EXAMPLES.

1. What is the discount of 1912 dollars 50 cents due 6 years hence, at 4½ per cent.?

```
        4,50
           3
        ‾‾‾‾‾
       13,50
      100,
```
         $. cts. $. cts.
$113,50 : 13,50   ::   1912,50 :
                              1350
                          ‾‾‾‾‾‾‾
                          95625
                          57375
                          19125
                          ‾‾‾‾‾‾‾‾‾ $. cts.
              1135,0)25818750,0(227,47+ Ans.
                     2270
                     ‾‾‾‾
                     3118
                     2270
                     ‾‾‾‾
                      8487
                      7945
                      ‾‾‾‾
                      5425
                      4540
                      ‾‾‾‾
                      8850
                      7945
                      ‾‾‾‾
                       905+

2. What is the present worth of 760 dollars due in 8 months, discount at 6 per cent. per annum?

```
              $.
8 mo. ⅔ 6 = 4
           100
           ‾‾‾
           104  :   100  ::  760 :
```
                 Ans. 730 dolls. 76 cts. 9 mills.

3. What is the present worth of 500 dollars payable in ¼ of a year, discount being at 5 per cent.?
                          Ans. $493 82⅔ cts.

4. A is to pay 592 dollars 70 cents on the first of April 1807, and 598 dollars 90 cents the first of July following. It is required to know how much money will discharge both sums on the first of January 1807, discounting at 8 per cent. per annum ?

Ans. $1156 94 cts. 3 mills.

5. Bought a quantity of goods for 500 dollars ready money, and sold them again for 666 dollars 67 cents, payable at $\frac{3}{4}$ of a year ; what was the gain in ready money, supposing discount to be made at 5 per cent. ?

Ans. $142 57 cts.

6. How much ready money will discharge a note for 150 dollars due in 60 days, allowing 6 per cent. per annum discount ?

Ans. $148 51 cts. 4 m+.

. . . . . .

EQUATION OF PAYMENTS.

EQUATION OF PAYMENTS is finding a time to pay at once several debts due at different times, so that no loss shall be sustained by either party.

RULE.* Multiply each payment by the time at which it is due ; then divide the sum of the products by the sum of the payments, and the quotient will be the time required.

---

* This rule is founded on a supposition, that the sum of the interests of the several debts which are payable before the equated time, from their terms to that time, is equal to the sum of the interests of the debts payable after the equated time, from that time to their terms ; but this is not correct, for by keeping a debt unpaid after it is due, the interest of it is gained for that time ; but by paying a debt before it is due, the payer does not lose the interest for that time, but the discount only, which is less than the interest.

Although this rule be not accurately true, yet in most questions that occur in business, the error is so trifling that it will be much used.

EXAMPLES.

1. A owes B 1900 dollars, to be paid as follows, viz. 500 dollars in 6 months, 600 dollars in 7 months, and 800 dollars in 10 months; what is the equated time to pay the whole debt ?

$$500 \times 6 = 3000$$
$$600 \times 7 = 4200$$
$$800 \times 10 = 8000$$

1900   )15200(8 months, Answer.
            15200

2. A owes B 240 dollars to be paid in six months ; but in 1½ month pays him 60 dollars, and in 4½ months after that 80 dollars more ; how much longer than six months should A in equity defer the rest ?   Ans. 3$\frac{9}{10}$ months.

3. I owe 6512 dollars, to be paid $\frac{1}{4}$ in 3 months, $\frac{1}{4}$ in 5 months, $\frac{1}{8}$ in 10 months, and the remainder in 14 months ; at what time ought the whole to be paid ?

Ans. 6$\frac{1}{4}$ months.

4. A owes 60 dollars to be paid in 90 days, 75 dollars in 60 days, and 50 dollars in 30 days ; what is the equated time for the whole to be paid ?   Ans. 61$\frac{6}{10}$ days.

. . . .

INVOLUTION.

INVOLUTION is the continual multiplication of a number into itself ; and the products thence arising, with the original number itself, are called the powers of that number.

Any number may itself be called a *first power*. If the first power be multiplied by itself, the product is called the *second power*, or square ; if the square be multiplied by the first power, the product is called the *third power*, or cube ; if the cube be multiplied by the first power, the product is called the *fourth power*, or biquadrate, &c.

Thus 3 is the first power of 3.
$3 \times 3 = 9$ is the second power of 3.
$3 \times 3 \times 3 = 27$ is the third power of 3.
$3 \times 3 \times 3 \times 3 = 81$ is the fourth power of 3, &c. &c.

And in this manner is formed the following table of powers.

TABLE of the Nine FIRST POWERS of the Nine DIGITS.

		1^1	2	3	4	5	6	7	8	9
Roots,	or 1st Pow.	1	2	3	4	5	6	7	8	9
Squares,	or 2d Pow.	1	4	9	16	25	36	49	64	81
Cubes,	or 3d Pow.	1	8	27	64	125	216	343	512	729
Biquadrates,	or 4th Pow.	1	16	81	256	625	1296	2401	4096	6561
Sursolids,	or 5th Pow.	1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes,	or 6th Pow.	1	64	729	4096	15625	46656	117649	262144	531441
Second Sursolids,	or 7th Pow.	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadr. Squar'd,	or 8th Pow.	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes Cubed,	or 9th Pow.	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489

The power of a number is often represented by a small figure, put above the number to the right, corresponding to the rank designed to be expressed; thus, $3,^1 3,^2 3,^3 3,^4 3,^5$ &c. respectively represent the first, second, third, fourth and fifth power of 3; and are respectively equal to 3, 9, 27, 81 and 243. These small figures are called indices or exponents of the powers. An exponent is never employed for the first power, except when it is to be compared with other powers.

RULE for raising the Powers of Numbers.

Multiply the given number or first power into itself, until the number be taken for a factor as many times as there are units in the index of the power to be found, and the last product will be the power required.

NOTE 1. The powers of fractions are found by raising each of their terms to the power required.

In every operation upon fractions, the fractions must be prepared, if necessary, in the manner formerly directed under multiplication of vulgar fractions.

An increase of the powers in proper fractions produces a decrease in their value; but the reverse in improper fractions.*

NOTE 2. If two or more powers are multiplied together, their product will be that power whose index is the sum of the exponents of the factors; thus $3 \times 3 = 9$, the square of 3; $9 \times 9 = 81$, the fourth power of 3; $81 \times 81 = 6561$, the eighth power of 3, &c.   EXAMPLES.

1. What is the 6th power of 8 ?

$$8$$
$$8$$
$$\overline{}$$
$$64 = \text{2d power.}$$
$$8$$
$$\overline{}$$
$$512 = \text{3d power.}$$
$$8$$
$$\overline{}$$
$$4096 = \text{4th power.}$$
$$8$$
$$\overline{}$$
$$32768 = \text{5th power.}$$
$$8$$
$$\overline{}$$
$$262144 = \text{6th power.} \qquad \text{Ans. } 8^6.$$

---

* The reason is plain; for the greater any number is, the greater will be its proportional increase under self multiplication.

2. What is the 7th power of $\frac{2}{3}$?

Ans. $\frac{128}{2187}$.

3. What is the 5th power of $\frac{9}{6}$ or $1\frac{1}{2}$?

Ans. $\frac{59049}{7776}$.

4. What is the 4th power of ,27 ?

Ans. ,0053144 1.

. . . .

EVOLUTION.

EVOLUTION is the reverse of involution, and teaches to find the root of any given powers.

The root of any power is such a number, as, being multiplied into itself a certain number of times, will produce *that* power.

NOTE 1. In a series of powers founded upon a given root, the evolution must always stop at the root ; for nothing can enter among the powers except the root and its multiples. Hence in the case of $\sqrt{256}$, we must stop at the root 16 ; though 16 has 2 for its own biquadrate root in a new series of powers of the number 2.

NOTE 2. The indices in evolution are similar to those in involution, and are placed in an angle of a character called a radical sign, which is put to the left of the power to be evolved, whether that power be a simple or compound number ; thus the third root of the simple number 50 is $\sqrt[3]{50}$, and the second root of it is $\sqrt{50}$, the index 2 being omitted, which index is always understood when a root is written without one. But if the power be a compound number, the radical sign is drawn over all parts of it ; thus the third root of 120+6 is $\sqrt[3]{120+6}$. The connecting line is called a vinculum.

And sometimes roots are designed like powers, with the reciprocal of the index of the root above the given number. So that the second root of 3 is $3^{\frac{1}{2}}$; the second root of 50 is $50^{\frac{1}{2}}$; and the third root of 50 is $50^{\frac{1}{3}}$; also the third root of 120+6 is $\overline{120+6}]^{\frac{1}{3}}$. This method of notation is used in algebra.

K

NOTE 3. A number is called a complete power of any kind when its root of the same kind can be accurately extracted ; but if not, the number is called an *imperfect* power, and its root a *surd* or *irrational* number ; so 4 is a complete power if it be considered as a square, its root being 2 ; but an imperfect power if it be considered as a cube, its root being a surd number.

NOTE 4. The power is first to be prepared for extraction or evolution, by dividing it into periods from the place of units to the left in integers, and to the right in decimal fractions ; each period containing as many places of figures as are denominated by the index of the root, provided the power contain a complete number of such periods. If it do not, the defect will be either on the right or left, or both. If the defect be on the right it may be supplied by annexing cyphers, and after this whole periods of cyphers may be annexed to continue the extraction if necessary. If there be a defect on the left hand, such defective period must remain unaltered, and is accounted the first period of the given number just the same as if it were complete.

This division is made by writing a point over the place of units, and over the last figure of every period on both sides of it ; that is, over every second figure if it be the second root, over every third if it be the third root, &c.

Thus, to point this number, 42176975,28061 ; for the

second root it will be   42176975,280610 ;

    But for the third root,   42176975,280610 ;

    And for the fourth,   42176975,28061000;

The root will contain just as many places of figures as there are periods or points in the given power ; and they will be integers or decimals respectively, as the periods are so from which they are found.

To extract the SQUARE ROOT.

RULE.* 1. Having distinguished the given number into periods, find a square number by the table or trial, either equal to or the next less than the first period, and put the root of it to the right hand of the given number, after the manner of a quotient figure in division, and it will be the first figure of the root required.

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* In order to show the reason of the rule, it will be proper to premise the following *Lemma.* The product of any two numbers can have at most but as many places of figures as are in both the factors, and, at least, but one less.

DEMONSTRATION. Take two numbers consisting of any number of places, but let them be the least possible of those places, viz. unity with cyphers, as 1000 and 100; then their product will be 1 with as many cyphers annexed as are in both the numbers, viz. 100000; but 100000 has one place less than 1000 and 100 together have; and since 1000 and 100 were taken the least possible, the product of any other two numbers, of the same number of places, will be greater than 100000; consequently the product of any two numbers can have, at least, but one place less than both the factors.

Again, take two numbers of any number of places that shall be the greatest of those places possible, as 999 and 99. Now 999×99 is less than 999×100; but 999×100 (=99900) contains only as many places of figures as are in 999 and 99; therefore 999×99 or the product of any other two numbers, consisting of the same number of places, cannot have more places of figures than are in both its factors.

COROLLORY 1. A square number cannot have more places of figures than double the places of the root, and, at least, but one less.

COR. 2. A cube number cannot have more places of figures than triple the places of the root, and, at least, but two less.

The truth of the rule may be shown algebraically thus :
Let N=the number whose root is to be found.
Now it appears from the lemma, that there will be always as many places of figures in the root as there are points or periods in the given number, and therefore the figures of those places may be represented by letters, viz.

2. Subtract the assumed square from the first period,
and to the remainder bring down the next period for a div-
idend.

3. Place the double of the root already found on the left
hand of the dividend for a divisor.

4. Consider what figure must be annexed to the divisor,
so that if the result be multiplied by it the product may be
equal to, or the next less, than the dividend, and it will be
the second figure of the root.

5. Subtract that product from the dividend, and to the
remainder bring down the next period for a new dividend.

6. Find a divisor as before, by doubling the figures al-
ready in the root ; and from these find the next figure of
the root as in the last article ; and so on through all the
periods to the last.

   To extract the Square Root of a Vulgar Fraction.

First prepare all vulgar fractions by reducing them to
their lowest terms, both for this and all other roots.   Then,

1. Take the root of the numerator and that of the de-
nominator for the respective terms of the root required ;
and this is the best way if the denominator be a complete
power.   But if not,

2. Multiply the numerator and denominator together ;
take the root of the product ; this root, being made the
numerator to the denominator of the given fraction, or the
denominator to the numerator of it, will form the fractional
root required.

Suppose N to consist of two periods, and let the figures in
the root be represented by a and b.

   Then $\overline{a+b}^2 = a^2 + 2ab + b^2 = N = $ given number ; and to
find the root of N is the same as finding the root of $a^2 + 2ab + b^2$, the method of doing which is as follows :

$$1\text{st divisor } a)a^2 + 2ab + b^2(a+b = \text{the root.}$$
$$a^2$$

$$2\text{d divisor } 2a+b)2ab+b^2$$
$$2ab+b^2$$

   Which operation exactly agrees with the rule, and the
same will be found to be true when N consists of any num-
ber of periods whatever.

3. Or reduce the vulgar fraction to a decimal, and extract its root.

EXAMPLES.

1. Required the square root of 6749604.

```
6749604(2598 Ans.
4
─────
45)274
 5)225
─────
509)4996
  9)4581
─────
5188)41504
   8)41504
```

2. Required the square root of 739,4.

```
739,40(27,19 + root.
4
─────
47)339
 7)329
─────
541) 1040
  1)  541
─────
5429)49900
   9)48861
─────
      1039 remainder.
```

3. What is the square root of 2 ?

Ans. 1,41421356+.

4. What is the square root of 10342656 ?

Ans. 3216.

5. What is the square root of 964,5192360241 ?

Ans. 31,05671.

6. What is the square root of ,00032754 ?

Ans. ,01809+.

7. What is the square root of $\frac{2733}{3300}$?   Ans. $\frac{7}{12}$.

K 2

8. What is the square root of $42\frac{1}{4}$?

Ans. $6\frac{1}{2}$.

9. What is the square root of $6\frac{2}{7}$?

Ans. 2,5298+ &c.

To extract the CUBE ROOT.

RULE.* 1. Having divided the given number into periods of 3 figures, find the nearest less cube to the first period by the table of powers or trial ; set its root in the quotient ; subtract the cube found from the first period, and to

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* The reason of pointing the given number, as directed in the rule, is obvious from Cor. 2, to the Lemma made use of in demonstrating the square root ; and the rest of the operation will be best understood from the following analytical process.

Suppose N, the given number, to consist of two periods, and let the figures in the root be denoted by a and b. Then $\overline{a+b}|^3 = a^3+3a^2b+3ab^2+b^3 = N = $ the given number ; and to find the cube root of N is the same as to find the cube root of $a^3+3a^2b+3ab^2+b^3$; the method of doing which is as follows :

$a^3+3a^2b+3ab^2+b^3(a+b=$ root.
a^3

$3a^2b+3ab^2+b^3$ resolvend.

$3a^2$
$\quad +3a$

$3a^2+3a$ divisor.

$3a^2b$
$\quad +3ab^2$
$\qquad +b^3$

$3a^2b+3ab^2+b^3$ subtrahend.

---

*   *   *

And in the same way may the root of any quantity consisting of any number of periods whatever, be found.

the remainder bring down the second period, and call this the resolvend.

2. To three times the square of the root just found, add three times the root itself, setting this one place more to the right than the former, and call this sum the *divisor*. Then divide the resolvend, omitting the unit figure, by the divisor, for the next figure of the root, which annex to the former, calling this last figure *e*, and the part of the root before found call *a*.

3. Add together these three products, viz. thrice the square of *a* multiplied by *e*, thrice *a* multiplied by the square of *e*, and the cube of *e*, setting each of them one place more to the right hand than the former, and call the sum the *subtrahend*, which must not exceed the resolvend ; but if it do, then make the last figure *e* less, and repeat the operation for finding the subtrahend.

4. From the resolvend take the subtrahend, and to the remainder join the next period of the given number for a new resolvend ; to which form a new divisor from the whole root now found, and thence another figure of the root as before, &c.

EXAMPLES.

1. Required the cube root of 48228,544.

$$3 \times 3^2 = 27 \quad | \quad 48228,544(36,4 \text{ root.}$$
$$3 \times 3 = 09 \quad | \quad 27$$

Divisor 279 | 21228 resolvend.

$$3 \times 3^2 \times 6 = \quad 162$$
$$3 \times 3 \times 6^2 = \quad 324 \quad \Big\} \text{Add.}$$
$$6^3 = \quad 216$$

$$3 \times 36^2 = 3888 \quad | \quad 19656 \text{ subtrahend.}$$
$$3 \times 36 = 108 \quad |$$

Divisor 38988 | 1572544 resolvend.

$$3 \times 36^2 \times 4 = \quad 15552$$
$$3 \times 36 \times 4^2 = \quad 1728 \quad \Big\} \text{Add.}$$
$$4^3 = \quad 64$$

1572544 subtrahend.

2. What is the cube root of 1092727 ?

Ans. 103.

3. What is the cube root of 34965783 ?

Ans. 327.

4. What is the cube root of ,0001357 ?

Ans. ,05138+.

5. What is the cube root of $\frac{1620}{3136}$?   Ans. $\frac{2}{5}$.

6. What is the cube root of $\frac{2}{3}$?   Ans. ,873+.

7. What is the cube root of 2 ?   Ans. 1,25+.

8. What is the cube root of $\frac{512}{729}$?   Ans. $\frac{8}{9}$.

To extract the ROOTS of POWERS in general.

RULE.* 1. Prepare the given number for extraction by pointing off from the place of units as the root required directs.

2. Find the first figure of the root by trial, and subtract its power from the given number.

3. To the remainder bring down the first figure in the next period, and call it the *dividend*.

4. Involve the root to the next inferior power to that which is given, and multiply it by the number denoting the given power, for a *divisor*.

5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

---

* This rule will be sufficiently obvious from the work in the following example. Extract the cube root of a^6+6a^5 $-40a^3+96a-64.$

$$a^6+6a^5-40a^3+96a-64(a^2+2a-4$$
$$a^6$$

$$3a^4)6a^5(+2a$$
$$\overline{\quad\quad\quad\quad} \quad\quad -3$$
$$a^6+6a^5+12a^4+8a^3=a^2+2a$$

$$-2$$
$$a^2+2a\times3=3a^4+12a^3+12a^2)-12a^4-48a^3+96a-64(-4$$
$$\overline{\quad\quad\quad\quad\quad}\quad\quad -3$$
$$a^6+6a^5-40a^3+96a-64=a^2+2a-4$$

$$*\quad\quad *\quad\quad *\quad\quad *\quad\quad *$$

6. Involve the whole root to the given power, and subtract it from the given number as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, and so on, until the whole is finished.

EXAMPLES.

1. What is the cube root of 53157376 ?

$$53157376(376 \text{ root.}$$
$$27 = 3^3$$

$$3^2 \times 3 = 27)261 \text{ dividend.}$$

$$50653 = 37^3$$

$$37^2 \times 3 = 4107)25043 \text{ second dividend.}$$

$$53157376 = 376^3$$

2. What is the biquadrate root of 34827998976 ?
Ans. 431,9+.

3. What is the sursolid root of 281950621875 ?
Ans. 195.

4. What is the square cubed, or sixth root of 16196,005 3044797,29 ? Ans. 5,03.

5. Find the seventh root of 34487717467,30751.
Ans. 32,01†.

6. Extract the eighth root of 7213895789838336.
Ans. 96.

7. What is the biquadrate root of 5308416 ?

$$5308416(48$$
$$256$$

$$4^3 \times 4 = 256)2748 \text{ dividend.}$$

$$5308416 = 48^4$$

ARITHMETICAL PROGRESSION.

Any rank of numbers increasing by a common excess, or decreasing by a common difference, are said to be in *Arithmetical Progression ;* such are the numbers 1, 2, 3, 4, &c. 7, 5, 3, 1 ; and ,8, ,6, ,4, ,2.   When the numbers increase they form an *ascending series ;* but when they decrease they form a *descending series.*

The numbers which form the series are called the *terms* of the progression.

Any *three* of the five following terms being given, the other two may readily be found.

 1st. The first term, } commonly called the *extremes.*
 2d. The last term, }
 3d. The number of terms.
 4th. The common difference.
 5th. The sum of all the terms.

PROBLEM I.

The first term, the last term, and the number of terms being given, to find the sum of all the terms.

Rule.* Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAMPLES.

1. The first term of an arithmetical progression is 1, the last term 21, the number of terms 11 ; required the sum of the series.

$$
\begin{array}{r}
21 \\
1 \\
\hline
22 \\
11 \\
\hline
2)242 \\
\hline
121
\end{array}
$$

Or $\dfrac{\overline{21+1}\times 11}{2} = 121$ the answer.

---

* Suppose another series of the same kind with the given one be placed under it in an inverse order ; then will the

2. How many strokes does a Venice clock strike in the compass of a day, going to 24 o'clock ?   Ans. 300.

3. If 100 stones be placed in a right line a yard distant from each other, and the first a yard from a basket ; what distance will that man go who gathers them up singly, returning with them one by one to the basket ?

Ans. 5 miles 1300 yards.

PROBLEM II.

The extremes and the number of terms being given, to find the common difference.

RULE.* Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference.

EXAMPLES.

1. The extremes are 3 and 19, and the number of terms is 9 ; required the common difference, and the sum of the whole series.

```
        9   19
        1    3
       ──   ──
     8) 16(
        ──
        2 difference, and  (19+ 3×9)/2 = 99 the sum.
```

---

sum of every two corresponding terms be the same as that of the first and last, consequently any one of those sums multiplied by the number of terms, must give the whole sum of the two series, and half that sum will evidently be the sum of the given series.

Let 1, 2, 3, 4, 5, 6, 7, be the given series.
And 7, 6, 5, 4, 3, 2, 1, the same inverted.

Then 8 + 8 + 8 + 8 + 8 + 8 + 8 = 56 or 8×7 and 1+2+3+4+5+6+7 = 56/2 = 28.

* The difference of the first and last terms evidently shows the increase of the first term by all the subsequent additions, until it becomes equal to the last ; and as the number of those additions is evidently one less than the number of terms, and the increase by every addition equal, it is plain that the total increase divided by the number of additions, must give the differences at every one separately ; whence the rule is manifest.

2. A man is to travel from Boston to a certain place in 12 days, and to go but three miles the first day, increasing every day by an equal excess, so that the last days journey may be 58 miles ; required the daily increase, and the distance of the place from Boston.

Ans. Daily increase 5, distance 366 miles.

3. A man had 12 sons whose several ages differed alike ; the eldest was 49, the youngest 5 years old ; what was the common difference of their ages ?

Ans. 4 years.

PROBLEM III.

Given the first term, the last term, and the common difference, to find the number of terms.

RULE.* Divide the difference of the extremes by the common difference, and the quotient increased by 1, is the number of terms required.

EXAMPLES.

1. The extremes are 3 and 19, and the common difference 2 ; what is the number of terms.

$$\begin{array}{r} 19 \\ 3 \\ \hline 2)16 \\ \hline 8 \\ 1 \\ \hline 9 \end{array}$$

Or $\dfrac{19-3}{2} + 1 = 9$ the answer.

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* By the last problem, the difference of the extremes divided by the number of terms less 1, gives the common difference ; consequently the same, divided by the common difference must give the number of terms less 1 ; hence this quotient, augmented by 1, must be the answer to the question.

In any arithmetical progression, the sum of any two of its terms is equal to the sum of any other two terms, taken at an equal distance on contrary sides of the former, or the double of any one term is equal to the sum of any two terms, taken at an equal distance from it on each side. The sum of any number of odd numbers (1, 3, 5, &c.) is equal to the square of that number.

2. A man travelled on a journey 5 miles the first day, and 35 miles the last day, increasing his journey every day by 3 miles ; how many days did he travel ?

Ans. 11 days.

3. Suppose a man travel the first day 7 miles, the last 51 miles, and increase his journey each day by 4 miles ; how many days will he travel, and how far ?

Ans. 12 days, and 348 miles.

. . . . . .

GEOMETRICAL PROGRESSION.

ANY series of numbers, the terms of which gradually increase or decrease by a constant multiplication or division, are said to be in *Geometrical Progression*. Thus, 4, 8, 16, 32, 64, &c. and 81, 27, 9, 3, 1, &c. are series in geometrical progression, the one increasing by a constant multiplication by 2, and the other decreasing by a constant division by 3.

The number by which the series is constantly increased or diminished, is called the ratio.

PROBLEM I.

Given the first term, the last term, and the ratio, to find the sum of the series.

RULE.* Multiply the last term by the ratio, and from the product subtract the first term, and the remainder divided by the ratio less 1, will give the sum of the series.

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* Take any series whatever, as 1, 3, 9, 27, 81, 243, &c. multiply this by the ratio, and it will produce the series 3, 9, 27, 81, 243, 729, &c.

Now let the sum of the proposed series be what it will, it is plain, that the sum of the second series will be as many times the former sum as is expressed by the ratio ; subtract the first series from the second, and it will give 729 —1 ; which is evidently as many times the sum of the first series, as is expressed by the ratio less 1 ; therefore

$\dfrac{729-1}{3-1}$=sum of the proposed series, and is the rule ; or

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EXAMPLES.

1. The extremes of a geometrical progression are 1 and 65536, and the ratio 4 ; what is the sum of the series ?

$$65536$$
$$4$$

$$262144$$
$$1$$

$$4-1=3)262143$$

$$87381 \text{ Ans. Or } \frac{4 \times 65536 - 1}{4 - 1} = 87381.$$

2. A man travelled 6 days ; the first day he went 4 miles, and doubling his travel each day, his last day's ride was 128 miles ; how far did he go in the whole.
                                                    Ans. 252 miles.

3. The extremes of a geometrical series are 1024 and 59049, and the ratio is $1\frac{1}{2}$; what is the sum of the series ?
                                                    Ans. 175099.

PROBLEM II.

Given the first term and the ratio, to find any other term assigned.

RULE.* Write down a few of the leading terms of the series, and place their indices over them, beginning with a cypher.

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729 is the last term multiplied by the ratio, 1 is the first term, and $3-1=2$ is the ratio less 1 ; and the same will hold let the series be what it will.

NOTE. Since in any geometrical series or progression, when it consists of 4 terms, the product of the extremes is equal to the product of the means ; and when it consists of 3 the product of the extremes is equal to the square of the mean ; it follows, that in any geometrical series, when it consists of an even number of terms, the product of the extremes is equal to the product of any two means equally distant from the extremes ; and when the number of terms is odd, the product of the extremes is equal to the square of the mean or middle term, or to the product of any two terms equally distant from them.

* DEMONSTRATION. In example first, where the first term is equal to the ratio, the reason of the rule is evident;

Add together the most convenient indices, to make an index less by 1 than the number expressing the place of the term sought. Multiply the terms of the geometrical series together, belonging to those indices, and make the product a dividend. Raise the first term to a power whose index is 1 less than the number of terms multiplied, and make the result a divisor. Divide the dividend by the divisor and the quotient will be the term sought.

NOTE. When the first term of the series is equal to the ratio, the indices must begin with an unit, and the indices added must make the entire index of the term required; and the product of the different terms, found as before, will give the term required.

EXAMPLES.

1. The first term of a geometrical series is 2, the number of terms 13, and the ratio 2 ; required the last term.

$$1, \ 2, \ 3, \ 4, \ 5, \ 6, \ 7, \text{ indices.}$$
$$2, \ 4, \ 8, \ 16, \ 32, \ 64, \ 128, \text{ leading terms.}$$

Then $6+7=$index to the 13th term.

And $64\times128=8192$ the answer.

In this example the indices must begin with 1, and such of them be chosen as will make up the entire index to the term required.

2. Required the 12th term of a geometrical series whose first term is 3, and ratio 2.

$$0, \ 1, \ 2, \ 3, \ 4, \ 5, \ 6, \text{ indices.}$$
$$3, \ 6, \ 12, \ 24, \ 48, \ 96, \ 192, \text{ leading terms.}$$

Then $6+5=$index to the 12th term.

And $192\times96=18432=$dividend.

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for as every term is some power of the ratio, and the indices point out the number of factors, it is plain from the nature of multiplication, that the product of any two terms will be another term corresponding with the index, which is the sum of the indices standing over those respective terms.

And in the second example, where the series does not begin with the ratio, it appears that every term after the two first, contains some power of the ratio multiplied into the first term, and therefore the rule in this case is equally evident.

The number of terms multiplied is 2, and 2—1=1 is the power to which the term 3 is to be raised ; but the first power of 3 is 3=divisor ; therefore 18432÷3=6144 the 12th term.

3. A young man agreed with a farmer to work for him 11 years with no other reward than the produce of one grain of wheat for the first year, allowing the increase to be tenfold, and that produce to be sowed the second year, and so on from year to year until the end of the time ; what is the sum of the whole produce, allowing 7680 grains to make a pint, and what does it amount to at one dollar fifty cents per bushel ?

Ans. 2260561 bushels, and 339084 dolls. 19 cts.

4. The first term of a geometrical series is 1, the ratio 2, and the number of terms 23 ; what is the last term ?
Ans. 4194304.

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ALLIGATION.

ALLIGATION teaches to mix several simples of different qualities, so that the composition may be of a middle quality ; and is commonly distinguished into two principal cases, called *Alligation Medial* and *Alligation Alternate*.

ALLIGATION MEDIAL.

Alligation Medial is the method of finding the rate of the compound, from having the rates and quantities of the several simples given.

RULE. Multiply each quantity by its rate ; then divide the sum of the products by the sum of the quantities, or the whole composition, and the quotient will be the rate of the compound required.

EXAMPLES.

1. Suppose 20 bushels of wheat at 10s. per bushel, 36 bushels of rye at 6s. per bushel, and 40 bushels of barley at 4s. per bushel were mixed together ; what would a bushel of this mixture be worth ?

$$20 \times 10 = 200$$
$$36 \times 6 = 216$$
$$40 \times 4 = 160$$

96   )576(6s, Answer.
         576

2. A composition being made of 5 pounds of tea at 7s.
per pound, 9 pounds at 8s. 6d. per pound, and 14½ pounds
at 6s. 10½d. per pound ; what is a pound of it worth ?
<div align="right">Ans. 7s. 4¾d+.</div>

3. A goldsmith mixes 8 pounds 5½ ounces of gold of 14
carats fine, with 12 pounds 8¼ ounces of 18 ; what is the
fineness of this mixture ?
<div align="right">Ans. 16 $\frac{81}{127}$ carats.</div>

4. If with 40 bushels of corn at 4s. per bushel, there are
mixed 10 bushels at 6s. per bushel, 30 bushels at 5s. per
bushel, and 20 bushels at 3s. per bushel ; what will 10 bush-
els of that mixture be worth ?
<div align="right">Ans. $7 16⅔ cts.</div>

5. A grocer would mix 12 cwt. of sugar at 10 dollars per
cwt. with 3 cwt. of 8⅔ dollars per cwt. and 8 cwt. at 7¼ dol-
lars per cwt. ; what will a cwt. of this mixture be worth ?
<div align="right">Ans. $8 95 cts. 6 mills.</div>

6. If 16 gallons of brandy at 1 dollar 25 cents, and 4 gal-
lons of water, be mixed with 40 gallons of wine at 3 dollars
per gallon ; what will the mixture be worth per gallon ?
<div align="right">Ans. $2 33¼ cts.</div>

ALLIGATION ALTERNATE.

Alligation Alternate is the method of finding what quan-
tity of any number of simples whose rates are given, will
compose a mixture of a given rate ; so that it is the re-
verse of alligation medial, and may be proved by it.

RULE.* Write the rates of the simples in a column un-
der each other.

* DEMONSTRATION. By connecting the less rate to the
greater, and placing the differences between them and the
mean rate alternately, the quantities resulting are such,
that there is precisely as much gained by one quantity as
is lost by the other, and therefore, the gain and loss upon
the whole are equal, and are exactly the proposed rate ;
and the same will be true of any other two simples mana-
ged according to the rule.

In like manner, let the number of simples be what it
may, and with how many soever each is linked, since it is
always a less with a greater than the mean price, there
<div align="center">L 2</div>

Connect a link with a continued line the rate of each simple, which is less than that of the compound, with one or any number of those, that are greater than the compound ; and each greater rate with one or any number of the less.

Write the difference between the mixture rate and that of each of the simples opposite the rates, with which they are respectively linked.

Then if only one difference stand against any rate it will be the quantity belonging to that rate ; but if there be several, their sum will be the quantity.

EXAMPLES.

1. A merchant would mix wines at 14s. 15s. 19s. and 22s. per gallon, so that the mixture may be worth 18s. per gallon ; what quantity of each must be taken ?

Or thus :

$$18\begin{cases}14\\15\\19\\22\end{cases}\quad\begin{matrix}4\text{ at }14s.\\1\text{ at }15s.\\3\text{ at }19s.\\4\text{ at }22s.\end{matrix}\text{Ans.}\qquad18\begin{cases}14\\15\\19\\22\end{cases}\quad\begin{matrix}1\ldots.\text{ at }14s.\\1+4=5\text{ at }15s.\\3+4=7\text{ at }19s.\\3\ldots.\text{ at }22s.\end{matrix}$$

2. How much corn at 2s. 6d. 3s. 8d. 4s. and 4s. 8d. per bushel, must be mixed together, that the compound may be worth 3s. 10d. per bushel ?

Ans. 12 at 2s. 6d. 12 at 3s. 8d. 18 at 4s. and 18 at 4s. 8d.

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will be an equal balance of loss and gain between every two, and consequently an equal balance on the whole.

It is obvious from the rule, that questions of this sort admit of a great variety of answers ; for having found one answer, we may find as many more as we please, by only multiplying or dividing each of the quantities found by 2, 3, or 4, &c. the reason of which is evident ; for, if two quantities of two simples make a balance of loss and gain with respect to the mean price, so must also the double or triple, the $\frac{1}{2}$ or $\frac{1}{3}$ part, or any other ratio of these quantities, *ad infinitum*.

Questions of this kind are called by algebraists *indeterminate* or *unlimited* problems, but by an analytical process, theorems may be raised that will give all the *possible* answers.

3. A goldsmith has gold of 18 carats fine, 16, 19, 22 and 24; how much must he take of each to make it 21 carats fine? Ans. 3 oz. of 16, 1 oz. of 18, 1 oz. of 19, 5 oz. of 22 and 5 oz. of 24 carats fine.

4. It is required to mix brandy at 80 cts. wine at 70 cts. cider at 10 cts. and water together, so that the mixture may be worth 50 cts. per gallon.

Ans. 9 gals. of brandy, 9 of wine, 5 of cider and 5 of water.

RULE 2. *When the whole composition is limited to a certain quantity.*

Find an answer as before by linking; then say, as the sum of the quantities, or differences thus determined, is to the given quantity, so is each ingredient found to the required quantity of each.

EXAMPLES.

1. How many gallons of water at 0 cts. per gallon, must be mixed with wine worth 60 cts. per gallon, so as to fill a cask of 100 gallons, and that a gallon may be afforded at 50 cts. ?

$$50 \begin{cases} 0 \rceil 10 \\ 60 \rfloor 50 \end{cases}$$

60 : 100 :: 10 :   60 : 100 :: 50 :
              10   50

6,0)100,0   6,0)500,0

16⅔   83⅓

Ans. 16⅔ gallons of water, and 83⅓ of wine.

2. How much wine at 80 cts. at 88 and 92 per gallon, must be mixed with 4 gallons at 75 cts. per gallon, so that the mixture may be worth 86 cts. per gallon ?

Ans. 4 gals. at 80 cts. 8½ at 88 and 8½ at 92.

3. How much gold of 15, of 17, of 18 and 22 carats fine, must be mixed together to form a composition of 40 ounces of 20 carats fine ?

Ans. 5 oz. of 15, 17 and 18, and 25 oz. of 22.

RULE 3. *When one of the ingredients is limited to a certain quantity.*

Take the difference between each price and the mean

rate as before ; then, as the difference of that simple whose quantity is given, is to the rest of the differences severally; so is the quantity given to the several quantities required.

EXAMPLES.

1. A grocer would mix teas at 1 dollar 20 cents, 66 cents and 1 dollar per pound, with 20 pounds at 40 cents per pound ; how much of each sort must he take to make the composition worth 80 cents per pound ?

$$
80 \left\{ \begin{array}{c} 40 \\ 66 \\ 100 \\ 120 \end{array} \right. \quad \begin{array}{l} 40 \\ 20 \\ 14 \\ 40 \end{array}
$$

lb.   lb.   lb.   lb.
40 : 20 :: 20 : 10 at 66 cts. ⎫
40 : 14 :: 20 : 7 at $1   ⎬ Ans.
40 : 40 :: 20 : 20   at   1 20 ⎭

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POSITION.

POSITION is a method of performing such questions as cannot be resolved by the common direct rules, and is of two kinds, *Single* and *Double.*

SINGLE POSITION.

Single Position teaches to resolve those questions whose results are proportional to their suppositions.

RULE.* 1. Take any number and perform the same operations with it as are described to be performed in the question.

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* Such questions properly belong to this rule, as require the multiplication or division of the number sought by any proposed number ; or when it is to be increased or diminished by itself, or any parts of itself, a certain proposed number of times. For in this case the reason of the rule is obvious ; it being then evident, that the results are proportional to the suppositions.

NOTE. 1 may be made a constant supposition in all questions ; and in most cases it is better than any other number.

2. Then say, as the result of the operation is to the position, so is the result in the question to the number required.

EXAMPLES.

1. A's age is double that of B, and B's is triple that of C, and the sum of all their ages is 140 ; what is the age of each ?

Suppose A's age to be 48
Then will B's$=\frac{48}{2}=24$
And C's$=\frac{24}{3}=$ 8

80 sum.

As 80 : 48 :: 140 : 84$=$A's age.
Consequently $\frac{84}{2}=42=$B's.
And $\frac{42}{3}=14=$C's.

140 Proof.

2. A certain sum of money is to be divided between 4 persons in such a manner that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{6}$ and the fourth the remainder, which is 28 dollars ; what is the sum ?

Ans. 112 dolls.

3. A person, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, had 60 dollars left ; what had he at first ?

Ans. 144 dolls.

4. What number is that which being increased by $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of itself, the sum will be 125 ?

Ans. 60.

5. A person lent his friend a sum of money, to receive interest for the same at 6 per cent. per annum, simple interest ; at the end of three years he received for principal and interest 383 dollars 50 cents ; what was the sum lent ?

Ans. 325 dolls.

6. A cistern is supplied with three cocks, A, B and C ; A can fill it in 1 hour, B in 2, and C in 3 ; in what time will it be filled by all of them together ?

Ans. $\frac{6}{11}$ hour.

D O U B L E   P O S I T I O N.

Double Position teaches to resolve questions by making two suppositions of false numbers.

RULE.* 1. Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Find how much the results are different from the result in the question.

3. Multiply each of the errors by the contrary supposition.

4. If the errors be alike, divide the difference of the products by the difference of the errors, and the quotient will be the answer.

5. If the errors be unlike, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

NOTE. The errors are said to be *alike,* when they are both too great or both too little ; and *unlike,* when one is too great and the other too little.

EXAMPLES.

1. A lady bought cambric for 40 cents a yard, and India cotton at 20 cents a yard ; the whole number of yards she bought was 8, and the whole cost 2 dollars ; how many yards had she of each sort ?

Suppose 4 yards of cambric, value $1 60 cts.
Then she must have 4 yards of cotton, value   80,

Sum of their values, 2 40
So that the first error is +40

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* This rule is founded on the supposition, that the first error is to the second as the difference between the true and the first supposed number is to the difference between the true and second supposed number ; when that is not the case, the exact answer to the question cannot be found by this rule.

NOTE. It will be often best to make 1 and 0 the suppositions.

Again, suppose she had 3 yards of cambric, $1 20 cts.
Then she must have 5 yards of India cotton, 1 00
                                                    ——————
                         Sum of their values, 2 20
                    So that the second error is + 20

Then 40—20=20=difference of the errors.
Also 4×20=80=product of the first supposition and
         second error.
And 3×40=120=product of the second supposition
      and first error.
And 120—80=40=their differences.
Whence 40÷20=2 yards of cambric, } Ans.
And 8—2=6 yards of India cotton, }

2. A and B have both the same income ; A saves ⅓ of his
yearly ; but B, spending 50 dollars a year more than A, at
the end of 4 years is 100 dollars in debt ; what is their in-
come, and what do they spend per annum ?

Ans. { Their income is $125 per year.
      { A spends $100.
      { B spends $150.

3. A laborer was hired for 40 days upon these conditions,
that he should receive 2 dollars for every day he wrought,
and forfeit 1 dollar for every day he was idle ; at the expi-
ration of the time he was entitled to 50 dollars ; how many
days did he work and how many was he idle ?

            Ans. He wrought 30 days and was idle 10.

4. A man had 2 silver cups of unequal weight, with one
cover for both, weight 5 oz ; now if he put the cover on the
less cup, it will be double the weight of the greater ; and
put on the greater cup, it will be three times the weight of
the less cup ; what is the weight of each cup ?

            Ans. 3 oz. the less, and 4 oz. the greater.

5. A person being asked what o'clock it was, answered
that the time past from noon was equal to ⅘ of the time
to midnight ; required the time.

                        Ans. 36 minutes past 12.

6. There is a fish whose head is 10 feet long ; his tail
is as long as his head and half the length of his body, and
his body is as long as his head and tail ; what is the whole
length of the fish ?

                              Ans. 80 feet.

7. A and B laid out equal sums of money in trade ; A gained a sum equal to ¼ of his stock, and B lost 225 dollars ; then A's money was double that of B's ; what did each lay out ?

Ans. $600.

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MISCELLANEOUS QUESTIONS.

1. A gentleman bought 27 yards of cloth at 2s. per yard, 24 yards at 3s. 1¼d. per yard, 25 yards at 1s. 8½d. per yard ; he also bought 3 yards of broadcloth, the price of which he does not recollect ; but on counting his money he found he had expended £.11 19s. 2¼d. ; what did his broadcloth cost per yard ?

Ans. $3 75 cts.

2. A servant went to market with £.5 and bought eggs at 7 for 4d. 2 pair of fowls at 2s. 4d. a pair, 17 pigeons at 3s. per dozen, 3 rabbits at 14d. each, and 3 dozen of larks at 14d. per dozen ; he also paid the baker £.2 17s. 1d. ; when he returned he had 21s. left ; how many eggs did he buy ?

Ans. 126.

3. I have a drawer 17 inches long, 12 inches broad and 7 inches deep ; how many one inch dice will it hold ?

Ans. 1428.

4. At a certain election 375 persons voted, and the candidate chosen had a majority of 91 ; how many voted for each ?

Ans. 233 and 142.

5. Suppose a man to step 30 inches at a time, and to go 4 miles an hour ; how many times does he step in a minute ?

Ans. 140⅘.

6. The divisor is 43967, the quotient 2737226, and the remainder 27672 ; what is the dividend ?

Ans. 120347643214.

7. A prize of $1000 is to be divided between two persons whose shares are in proportion of 7 to 9 ; required the share of each ?

Ans. { $437 50 cts. { $562 50 cts.

8. After paying away $\frac{1}{4}$ and $\frac{1}{3}$ of my money, I had 66 guineas left in my purse ; what was in it at first ?

Ans. 120.

9. A reservoir for water has two cocks to supply it ; the first alone will fill it in 40 minutes, the second in 50 minutes ; and it has a discharging cock by which it may be emptied when full, in 25 minutes. Now supposing that these three cocks are all opened, that the water comes in, and that the influx and eflux of the water are always a-like, in what time would the cistern be filled ?

Ans. 3 hours 20 minutes.

10. A gentleman went to 4 taverns in succession ; upon entering each of them he borrowed as much money as he carried to it, and upon leaving them, he paid each landlord one dollar ; which done he finds himself without money : What sum did he carry to the first tavern ?

Ans. 93 cts. 7$\frac{1}{2}$ mills.

11. If to my age there be added its half and fifth, and 35 with its half and fifth, the sum will be 102 ; what is my age ?

Ans. 25.

12. In a mixture of wine cider, $\frac{1}{2}$ of the whole added to 25 gallons, was wine ; and $\frac{1}{3}$ part, less 5 gallons, was cider ; how many gallons were there of each ?

Ans. 85 of wine and 35 of cider.

13. A hare is fifty of her own leaps before a greyhound, and takes 4 leaps to the greyhound's 3 ; but 2 of the greyhound's leaps are as much as 3 of the hare's ; how many leaps must the greyhound take to catch the hare ?

Ans. 300.

14. Out of a cask of wine which had leaked away $\frac{1}{3}$ part, 21 gallons were drawn ; and then being gauged, it was found to be half full ; how many gallons did it hold ?

Ans. 126.

15. What part of 4d. is $\frac{4}{9}$ of 6 pence ?

Ans. $\frac{2}{3}$.

16. What number is that from which, if 5 be subtracted $\frac{2}{3}$ of the remainder is 80 ?

Ans. 125.

17. A post is $\frac{1}{4}$ in the mud, $\frac{1}{4}$ in the water, and 10 feet above the water ; what is its whole length ?

Ans. 24.

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18. The air presses in fair weather upon a human body about 33905 pounds ; and in foul weather but 30624 pounds ; what difference of weight lies on such a body in these alterations of the weather ?

Ans. 3281.

19. Hipparchus and Archimedes of Syracuse, flourished about 200 years before Christ ; Possidonius 50 years before, and Ptolemy 140 years after Christ ; all advanced the science of Astronomy ; how long did each of these persons flourish before the year 1807 ?

Ans. Hipparchus and Archimedes 2007, Possidonius 1857, and Ptolemy 1667 years before Christ.

20. A is 17, B 7 years of age ; what will their ages severally be when the elder is double the age of the younger ?

Ans. { A will be 20,
{ B 10.

21. A sheepfold was robbed three nights successively ; the first night half the sheep were stolen and half a sheep more ; the second night half of the remainder were lost, and half a sheep more ; the last night they took half what were left, and half a sheep more, by which time they were reduced to 20 ; how many were there at first ?

Ans. 167.

22. What is the difference and what the sum of six dozen dozen and half a dozen dozen ?

Ans. { The sum is 936.
{ The difference 792.

23. What difference is there between twice eight and twenty, and twice twenty eight ; as also between twice five and fifty, and twice fifty five ?

Ans. 20 and 50.

24. The continual multiplication of the nine digits will give the number of changes that may be rung on 9 bells ; how many changes are there ?

Ans. 362880.

25. What number is that to which if $\frac{3}{10}$ of $\frac{16}{7}$ of $\frac{141}{213}$ be added, the total will be one ?

Ans. $\frac{3648}{7455}$.

26. If $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ship be worth $\frac{1}{9}$ of $\frac{5}{7}$ of $\frac{11}{13}$ of the cargo, valued at 40000 dollars ; what is the value of ship and cargo ?

Ans. $50744 81 cts.

27. A grocer would mix a quantity of sugar at 10d. per pound, with other sugars at $7\frac{1}{3}$d. 5d. and $4\frac{1}{2}$d. per pound, intending to make up a compound worth 6d. per pound ; what quantity of each must he take ?

Ans. $1\frac{1}{3}$ lb. at 10d. 1 lb. at $7\frac{1}{2}$d. $1\frac{1}{2}$ lb. at 5d. and 4 lb. at $4\frac{1}{2}$d.

28. Four nines may be so placed, as to denote and be read for 100,; how is it done ?

29. Sound, not interrupted, is found by experiment to move uniformly about 1150 feet in a second of time ; how long then, after firing an alarm gun at Fort Independence, may the same be heard at Cambridge, taking the distance at $5\frac{2}{3}$ miles ?

Ans. $26\frac{2}{6}\frac{4}{0}$ seconds.

30. If I see the flash of a gun fired by a vessel in distress at sea, which happens we will suppose at the instant of its going off, and hear the report a minute and 3 seconds afterwards ; how far is she off ?

Ans. 72450 feet.

31. An elm plank is 14 feet 3 inches long ; what distance from the edge must a line be struck to take off a yard square ?

Ans. $7\frac{11}{18}$ inches.

32. A man dying left his wife in expectation that a child would be afterward added to the family, and in making his will ordered, that if the child were a son, $\frac{2}{3}$ of his estate should belong to him, and the remainder to his mother ; but if it were a daughter, he appointed the mother $\frac{2}{5}$ and the child the remainder ; but it happened that the addition was both a son and a daughter, by which the widow lost in equity 2400 dollars more than if there had been only a girl ; what would have been her dowry, had she have had only a son ?

Ans. \$2100.

. . . .

MEASUREMENT OF GRINDSTONES.

GRINDSTONES are sold by the stone, and their contents found as follows :*

---

* 24 Inches in diameter, and 4 inches thick, make a stone.

RULE. To the whole diameter add half of the diameter, and multiply the sum of these by the same half, and this product by the thickness ; divide this last number by 1728, and the quotient is the contents, or answer required.

EXAMPLES.

1. What are the contents of a grindstone 24 inches diameter and 4 inches thick ?

```
                24 diameter.
                12 half diameter.

                ──
                36
                12
                ──
                432
                    4 thickness.
      ────────────────────────
      1728)1728
      ────────────────────────
```

Ans. 1 stone.

2. What are the contents of a grindstone 36 inches diameter and 4 inches thick ?

```
                36
                18
                ──
                54
                18
                ──
                432
                54
                ───
                972
                4
      ───────────────
      1728)3888(2¼
           3456
      ───────────────
            432
            4
      ───────────────
      1728)1728(1
           1728
                ── Ans, 2¼ stone.
```

BOOK KEEPING.

———◆———

Book KEEPING is a branch of science which teaches to record mercantile transactions in a regular and systematic manner.

A merchant's books should exhibit the true state of his affairs, and the particular success of each transaction as well as the general result of the whole ; and should afford a correct and ready information upon every subject for which they may be consulted.

Books may be kept either by *single* or by *double* entry. Double entry being used only in extensive mercantile affairs, does not fall within the compass of the design of this work, and is therefore not considered. Single entry is chiefly used by traders in retail business, and by farmers and mechanics, being the most simple and concise method of Book keeping. It teaches to record transactions on credit, and for this purpose two books are required, the Day Book and the Ledger.

The Day Book begins with an account of the debts due to the owner, and those due from him to others ; then follows a detail of the occurrences of trade, set down in the order of time in which they happened, with all the necessary circumstances of time, quantity, price, conditions and all other particulars, which may serve to render the entries, whenever referred to, satisfactory and intelligible.

If any dispute happen in trade, this book is produced as the principal voucher ; every transaction recorded in it should, therefore, be stated with care and accuracy.

The Day Book is ruled with a margin on the left hand, and two columns on the right for *Dolls. cts.* On the top of each page is written the name of the town, and date of the account that follows. The name of the person or customer is written over each account with the term Dr. or Cr annexed, according as he becomes debtor or creditor

M 2

the transaction, which may be known by the following rule.

The person who receives is debtor, and the person who gives or parts with any thing is creditor.

Thus, if I sell goods on credit, I enter A B *(the buyer)* Dr. to the goods, specifying the quantity and the value. If I buy goods on credit I enter C D *(the seller)* Cr. by the goods, specifying their quantity and value. By the same rule, if I pay money, the person to whom I pay it is made Dr. to cash for the amount paid, and if I receive money, the person from whom I receive it is made Cr. by cash for the amount received ; and when debts are contracted or discharged by any other means, the same rule is observed ; the person who becomes indebted to me is entered Dr. and the person to whom I become indebted is entered Cr.

The Ledger collects together the dispersed accounts of each person in the Day Book, and places the Dr. and Cr. on opposite pages of the same folio. The persons name is written in large characters on the top of the account ; on the left hand page or side he is styled Dr. on the opposite side or right hand page Cr. On these pages the transactions are entered as they stand Dr. or Cr. in the Day Book ; each person with whom I have dealings, is debited for what he has bought of me on credit, and on the opposite page he is credited for all the payments he has made, and the difference between these two sides is called the balance.

In order the more readily to turn to any account in the Ledger, an *index* is always prefixed, to contain the name of each debtor or creditor, and the page on which his account is posted ; and as soon as an account is opened in the Ledger, the name and page are entered in the index in alphabetical order ; and the account is then marked ($\rightleftharpoons$) on the margin of the Day Book to show that it is posted ; and every following transaction with the same person is posted on its proper side under his name already written in the Ledger, as an account is never opened twice with the same person, though it may be transfered from one folio to another for want of room.

The Ledger is ruled in a folio form with a margin for the date on the left hand, and two columns for *Dolls. cts.* on *the right, and* next to the money column one for reference *to the Day Book.*

When two or more persons or things are included in the same account, they are expressed by the term *sundries*.

Example.

Suppose David Davis owes me 450 dollars for the balance of an account with him, April 1st, 1807 ; the next day April 2d, I buy of him 200 bushels of wheat at 1 dollar 50 cents per bushel, and 100 bushels of corn at 75 cents per bushel ; the next day, April 3d, I sell Jonathan Worth 150 bushels of wheat at 1 dollar 75 cents per bushel ; April 4th, Jonathan Worth pays me 200 dollars in cash, and David Davis pays me 50 dollars in cash ; required the Day Book and Ledger of the transaction.

DAY BOOK, No. 1.

Hallowell, April 1, 1807.

		Dolls.	cts.
David Davis,	Dr.		
To balance due on old account, . . .		450	00
————April 2.————			
David Davis	Cr.		
By 200 bushels wheat, . . . at $1,50		300	00
100 do. corn, ... 75		75	00
		375	00
————April 3.————			
Jonathan Worth	Dr.		
To 150 bushels wheat, . . . at 1,75		262	50
————April 4.————			
Jonathan Worth	Cr.		
By cash in part for wheat, ...		200	00
David Davis	Cr.		
By cash fifty dollars, ...		50	00

To post the above accounts, open an account for David Davis, debit him for 450 dollars ; and for the second days transaction credit him for 375 dollars ; for the third open an account for Jonathan Worth, debiting him 262 dollars 50 cents ; and for the fourth day credit him for 200 dollars, and credit David Davis for 50 dollars.

LEDGER No. 1.

1807.	*Dr.*	David Davis,		Dolls.	Cts.
April 1.	To balance of old account,		139	450	00
				450	00
April 4.	To balance of above account,		—	25	00

1807.	*Dr.*	Jonathan Worth,			
April 3.	To wheat, ... , .		139	262	50
				262	50
4.	To balance of old account,		—	62	50

LEDGER No. 1.

1807.	*Contra*		*Cr.*		Dolls.	Cts.
April 2.	By sundries, ...		139		375	00
4.	By cash, ...		–		50	00
	By balance to new account,		–		25	00
					450	00

1807.	*Contra*		*Cr.*			
April 4.	By cash, ...		139		200	00
	By balance to new account,		–		62	50
					262	50

* Opposite pages in the Ledger are both numbered like.

By the foregoing Ledger it appears that the balances are
in my favor, which, added to the cash I have on hand, and
the goods unsold, show the amount of my stock, which com-
pared with my original stock, will show my profit or loss,
viz.

		Dolls.	cts.
David Davis owes me		25	00
Jonathan Worth do.		62	50
I have in cash,		250	00
Wheat unsold 50 bushels, valued at prime cost $1,50	}	75	00
Corn do. 100 bushels do. at 75 cts.		75	00
Amount of my stock,		487	50
My original stock was		450	00
I have therefore gained		37	50

NOTE. If you should enter any thing in your Ledger
under a wrong title, or in any other way false, it should
not be blotted out, but marked thus (×) in the margin
against it; and write on the opposite side *Error per contra*,
with the sum against it, and make the same mark in the
margin.

DAY BOOK NO. 2.

Hallowell, April 6, 1807.

		$.	cts.
George Simpson	Dr.		
To balance due on old account,		200	
David Holland	Dr.		
To balance due on old account,		150	
John Barton	Dr.		
To balance due on account of 6 Hhds. } W. I. Rum,		340	
William Read	Cr.		
By balance due him on account of } English goods pr. invoice,		462	44
Daniel Cushing	Cr.		
By balance due him on account,		76	60
Thomas Tilton	Cr.		
By balance due him for 6 months } service on farm,		90	
————April 8.————			
Charles Prince	Dr.		
To 1 2 17½ lb. sugar, at $12		19	88
2 bbls. superfine flour, at 7,50		15	
1 do. mess pork,		25	
		59	88
Richard Lewis	Dr.		
To 2 yds. superfine broadcloth, at $6		12	
4 — cassimere, best blk. 15s.		10	
1½ doz. buttons, 5s. 6d.		1	37
1 do. small do.			38
4 skeins silk, 4½d.			25
4 sticks twist, 4½d.			25
		24	25
	Cr.		
By cash in part,		15	00

Hallowell, April 9, 1807.

			$.	cts.
James Athearn, Esq.	*Dr.*			
To cask Lisbon wine, 41 gals. at	$2		82	
10 bbls. cider,	3,50		35	
(Payable in 30 days.)				
			117	

----April 10.----

Thomas Nye	*Dr.*			
To 6½ yds. calico,	at 39 cts.		2	53

Jonathan Drew	*Dr.*			
To 3½ yds. velvet,	at 6s. 9d.		3	94
⅞ — Marseilles vesting,	10s. 8d.		1	33
1¾ — blue broadcloth,	20s.		5	83
2 — cotton cambric,	3s. 6d.		1	17
4 — platillas,	2s. 6d.		1	67
			13	94

----April 13.----

Tristram Eastman,	*Dr.*			
To 2 gals. rum,	at 96 cts.		1	92
4 — molasses,	50		2	
2 lb. souchong tea,	1,00		2	
6½ loaf sugar,	30		1	95
			7	87

Thomas Tilton	*Dr.*			
To cash in part of the balance due him,			50	
his order paid Samuel Lane,			21	50
			71	50

----April 14.----

George Mathews	*Dr.*			
To 4 M 10d. nails,	at 12s.		8	
10 — 4d. do.	at 2s. 6d.		4	17
			12	17

George Simpson	*Cr.*			
By cash (per rect.)			41	

Hallowell, April 16, 1807.

			S.	cts.
Nehemiah Gordon		Dr.		
To 2 pieces India cottons, 20 yards each,	} at 26s.		8	67
1½ yd. Marseilles vesting,	8s.		2	
1 lb. bohea tea,				46
			11	13
Daniel Cushing		Dr.		
To cash (pr. receipt)			20	
10 lb. cotton,	at 2s.		3	33
4 cwt. 2 qrs. Swedes iron,	$8		36	
14 lb. sugar,			2	
2 gals. rum,	96 cts.		1	92
			63	25
———————April 18.———				
Arthur Johnson		Dr.		
To 6 lb. coffee,	at 2s.		2	
4 lb. candles,	20 cts.			80
			2	80
Hyde & Parkman		Dr.		
To hhd. W. I. rum, 3d proof, 112 gals.				
3 out.				
——— cts.				
109	at 86		93	74
Nehemiah Gordon		Cr.		
By cash received of John Thomas,			5	
———————April 21.———				
Rufus Allen		Dr.		
To 4⅖ bushels salt,	at 4s. 6d.		3	38
1 lb. souchong tea,			1	
1 — pimento,				42
½ — pepper,	at 3s.			25
4 — tobacco,	at 1s. 6d.		1	
			6	05

N

Hallowell, April 21, 1807.

			$.	cts.
	Samuel King *Dr.*			
=	To 1 gal. N. E. rum,			67
	7 lb. sugar,		1	
	1¼ lb. powder,	at 2s. 6d.		62
	3 - shot,	1s.		50
			2	79
	———April 24.———			
	Benjamin Foster *Dr.*			
=	To 1 lb. indigo,		3	50
	Richard Lewis *Dr.*			
	To 5 pieces India cottons,	at 26s.	21	67
=	7 yds. cotton cambric,	4s. 6d.	5	25
	6 - col'd do.	3s.	3	
			29	92
	———April 25.———			
	David Holland *Cr.*			
=	By cash, (received by mail)		96	
	George Simpson *Cr.*			
=	By check on Hallowell & Augusta Bank } for one hundred & fifty dollars, }		150	
	———April 30.———			
	John Barton *Cr.*			
=	By cash rec'd for 25 bbls. beef sold at } $10 50 cts. }		262	50
	Thomas Nye *Dr.*			
	To 3 bushels salt,	at 4s. 6d.	2	25
=	3 gals. molasses,	3s.	1	50
	1 M 4d nails,			42
			4	17
	———May 2.———			
	Charles Prince *Cr.*			
=	By check on Union Bank, Boston, for } one hundred dollars, }		100	
	Dr.			
=	To his order paid James Holman for } forty dollars, }		40	

Hallowell, May 2, 1807.

		$.	cts.
Daniel Cushing Dr.			
To 2 pieces calico, 28 & 28½ yds. at 30 cts.		16	95
8 yds. satinnet, at 15s.		20	
2 - broadcloth, $4		8	
1½ - vesting, 8s.		2	
1¼ doz. gilt coat buttons, 5s. 6d.		1	15
		48	10

————————May 6.————————

James Athearn, Esq. Dr.			
To 60 yds. India carpeting, at 90 cts.		54	

Rufus Allen Cr.			
By cash,		5	

————————May 9.————————

Jonathan Drew Dr.			
To 7 lb. sugar,		1	
2 gals. rum, at 96 cts.		1	92
4 lb. tobacco, 1s. 6d.		1	
		3	92
Cr.			
By 116 lb. butter, at 1s.		19	33

James Athearn, Esq. Cr.			
By cash, one hundred & twenty five dols.		125	

————————May 12.————————

William Read Dr.			
To cash inclosed in a letter, forwarded } by J. Jones,		500	

Thomas Tilton Dr.			
To cash paid him to balance his account,		18	50

————————May 16.————————

William Read Cr.			
By pipe brandy, 126 gals. at $1		126	
40 bbls. fine flour, 6		240	
10 - superfine do. 6 50		65	
		431	

Hallowell, May 16, 1807.

			S.	cts·
Arthur Johnson		Dr.		
To 1 gal. Lisbon wine,			2	25
½ do. gin,	at 7s. 6d.			63
6 lb. 12 oz. loaf sugar,	30 cts.		2	02
			4	90
		Cr.		
By 10 bushels corn,	at 4s. 6d.		7	50

————————May 18.————————

Charles Prince		Dr.		
To 5 bbls. superfine flour,	at $8		40	00

Richard Lewis		Dr.		
To 14 yd's col'd cambric,	at 3s.		7	00
7 -- linen,	5s. 6d.		6	42
1 oz thread, No. 40				42
2 pair Morocco shoes,	6s. 6d.		2	17
1 oz indigo,				25
			16	26

————————May 25.————————

James Athearn, Esq.		Dr.		
To his order paid T. Noyes for goods,			67	62

Charles Prince		Dr.		
To his order paid T. Nye,			10	50

Thomas Nye		Cr.		
By order from C. Prince,			10	50
		Dr.		
To 3½ yds. coating,	at 10s.		5	83

————————May 26.————————

Jonathan Drew		Dr.		
To box 7 by 9 glass,			16	
4 M 10d nails,	at $2		8	
6 - 6d do.	,75		4	50
10 - 4d do.	,42		4	20
			32	70

Hallowell, May 27, 1807.

			$.	cts.
Tristram Eastman	*Dr.*			
To 6 bushels corn,	at ,90		5	40
2 do. salt,	,75		1	50
			6	90

————————May 30.————————

		$.	cts.
George Mathews	*Dr.*		
To 2 bbls. superfine flour,	at $8	16	
	Cr.		
By cash, ten dollars,		10	

		$.	cts.
Arthur Johnson	*Dr.*		
To 6 lb. raisins,	at 1s.	1	
7 lb. sugar,		1	
¼ lb. pepper, 1s. 2 oz. cinnamon, at 6d.			33
		2	33

————————June 1.————————

		$.	cts.
Jonathan Drew	*Cr.*		
By cash to balance his account,		31	23

		$.	cts.
David Holland	*Dr.*		
To 4 gals. brandy,	at $1,25	5	
keg for do.			62
		5	62

		$.	cts.
George Simpson	*Cr.*		
By cash to balance his account,		9	

		$.	cts.
Thomas Nye	*Dr.*		
To 7 yds. cotton chambray,	at 3s. 6d.	4	08
2 pieces India cottons, 20 yds. ea. at 24s. 6d.		8	17
2 fancy shawls,	at 6s. 8d.	2	22
14 yds. calico,	2s. 6d.	5	83
1 oz. No. 38 thread,			38
4 skeins silk,	at 4½d.		25
14 yds. linen,	at 4s. 6d.	10	50
		31	43

N 2

Hallowell, June 3, 1807.

			$.	cts.
	Thomas Nye	*Cr.*		
=	By 2 days work in garden,	at $1,00	2	
	Charles Prince	*Dr.*		
	To 2 loaves sugar, 6 ℔. 11 oz. & 6 lb. } 5 oz.　at 30 cts. }		3	90
=				
	2 lb. hyson skin tea,	at 92 cts.	1	84
	6 lb. coffee,	at 2s. 3d.	2	25
			7	99
	George Mathews	*Dr.*		
=	To 2 bushels corn,	at 90 cts.	1	80
	12 lb. butter,	20	2	40
			4	20
	Nehemiah Gordon	*Dr.*		
≠	To 4 lb. bohea tea,	at 2s. 9d.	1	83
	————June 4.————			
	William Read	*Dr.*		
	To keg butter, 35			
	3 tare.			
=	———			
	32 lb.	at 20 cts.	6	40
	keg for do.			42
	(Shipped per Packet.)			
			6	82
	Hyde & Parkman	*Dr.*		
	To hhd. sugar,　9 3 21 lb.			
=	Tare 12 pr. ct. 1 0　7			
	———			
	8 3 14	at $11	97	62
		Cr.		
=	By 11520 feet mercht. boards,	at $10	115	20
	Nehemiah Gordon	*Cr.*		
=	By cash to balance his account,		7	96

Hallowell, June 6, 1807.

	Dr.	$.	cts.
Samuel King			
To 4 bushels salt, at ,75		3	
1½ gal. N. E. rum, 4s.		1	
		4	
	Cr.		
By transfer of the balance of his account to Rufus Allen, per order;		6	79

	Dr.		
Benjamin Foster			
To 1 handsaw,		1	88
2 plane irons, 2s. 3d. & 1s. 6d.			62
1 chisel,			25
5 screw augers, ¾, ⅘, ⅚, ⅞ & ⅞, 25 qrs. at 9d.		3	12
		5	87

	Dr.		
Rufus Allen			
To Samuel King's account, transfered per order,		6	79
¼ cwt. sugar, at $15,50		3	88
2 gals. brandy, 1,25		2	50
7 yds. calico, 2s. 1d.		2	43
		15	60

	Dr.		
Charles Prince			
To 10 gals. brandy, at 1,20		12	

	Cr.		
Richard Lewis			
By his note, payable in 30 days,		55	43

	Dr.		
William Read			
To cash to balance his account,		386	62

INDEX *to* LEDGER No. 2.

1807.	*Dr.*	*George Simpson,*	Fol.	Dolls.	Cts.
April 6.	To balance, . . , ...		1	200	00
				200	00

1807.	*Dr.*	*David Holland,*			
April 6.	To balance, ...		1	150	00
June 1.	To sundries, , ...		7	5	62

LEDGER No. 2.

1807.	Contra	Cr.	Fol.	Dolls.	Cts.
April 14. By cash, ...			2	41	00
25. By check on H. & A. Bank, .			4	150	00
June 1. By cash, . , ...			7	9	00
				200	00

1807.	Contra	Cr.			
April 25. By cash, ...			4	96	00

1807. Dr. John Barton, (Bath.)

			Dolls.	Cts.
April 6.	To balance for rum, ...	1	340	00
×June 4.	To butter, , . , ＋ . . .	8	6	82

1807. Dr. William Read,

May 12.	To cash, ...	5	500	00
June 6.	To butter omitted June 4, 1807,	8	6	82
	To cash, ...	9	386	62
			893	44

1807. Dr. Daniel Cushing,

April 16.	To sundries, ... , - ,	3	63	25
May 2.	To sundries, . . . , . . .	5	48	10

1807.	*Contra*	*Cr.*		Dolls.	Cts.
April 30. By cash, ...			4	262	50
✕ June 6. By error in posting, . . .				6	82

1807.	*Contra*	*Cr.*			
April 6. By balance due, ...			1	462	44
May 16. By sundries, ...			5	431	00
				893	44

1807.	*Contra*	*Cr.*			
April 6. By balance due, , ...			1	76	60

1807.　　*Dr. Thomas Tilton,*

		Dolls.	Cts.
April 13. To sundries, ...	2	71	50
May 12. To cash, ...	5	18	50
		90	00

1807.　*Dr.*　　*Charles Prince,*

April 8. To sundries, ...	1	59	88
May 2. To his order, ...	4	40	00
18. To flour, ...	6	40	00
25. To his order, ...	6	10	50
June 3. To sundries, ...	8	7	99
Transferred to folio 7,		158	37

1807.　*Dr. Richard Lewis,* (York.)

April 8. To sundries, ...	1	24	25
24. To sundries, ...	4	29	92
May 18. To sundries, ...	6	16	26
		70	43

1807.	*Contra*	*Cr.*		Dolls.	Cts.
April 6. By balance,			1	90	00
				90	00

1807.	*Contra*	*Cr.*			
May 2. By Union Bank,			4	100	00
By balance transferred to folio 7,				58	37
				158	37

1807.	*Contra*	*Cr.*			
April 8. By cash,			1	15	00
June 6. By note to balance,			9	55	43
				70	43

1807. *Dr. James Athearn, Esq.*

			Dolls.	Cts.
April 9.	To sundries, ...	2	117	00
May 6.	To carpeting, ...	5	54	00
25.	To order, ... , . .	6	67	62

1807. *Dr. Thomas Nye,* (Athens.)

April 10.	To calico, ...	2	2	53
30.	To sundries, ...	4	4	17
May 25.	To coating, ...	6	5	83
June 1.	To sundries, , ...	7	31	43

1807. *Dr.* . . . *Jonathan Drew,*

April 10.	To sundries, ...	2	13	94
May 9.	To sundries, ...	5	3	92
26.	To sundries, ...	6	32	70
			50	56

1807.	*Contra*	*Cr.*		Dolls.	Cts.
May 9.	By cash, ...	5	125	00

1807.	*Contra*	*Cr.*			
May 25.	By order, ...	6	10	50
June 3.	By work, ...	8	2	00

1807.	*Contra*	*Cr:*			
May 9.	By butter, ...	5	19	33
June 1.	By cash, ...	7	31	23
			50	56

O 2

1807. *Dr.*　　*Tristram Eastman,*

			Dolls.	Cts.
April 13.	To sundries, ...	2	7	87
May 27.	To corn & salt, ...,	7	6	90

1807. *Dr.*　　*George Mathews,*

			Dolls.	Cts.
April 14.	To nails, . . , ...	2	12	17
May 30.	To flour, ...	7	16	00
June 3.	To sundries, . . , ...	8	4	20

1807. *Dr.*　　*Nehemiah Gordon,*

			Dolls.	Cts.
April 16.	To sundries, ...	3	11	13
June 3.	To tea, ...	8	1	83
			12	96

1807.	*Contra*	*Cr.*		Dolls.	Cts.

1807.	*Contra*	*Cr.*			
May 30. By cash, ...			7	10	00

1807.	*Contra*	*Cr.*			
April 18. By cash, ...			3	5	00
June 4. By cash, . . , ...			8	7	96
				12	96

1807.	*Dr.*	*Arthur Johnson,*		Dolls.	Cts.
April 18.	To sundries, . . . , . . . ,		3	2	80
May 16.	To sundries, . . , . . . :		6	4	90
30.	To sundries, . . . : , :		7	2	33

1807.	*Dr.*	*Hyde* & *Parkman,*			
April 18.	To rum, ... ,		3	93	74
June 4.	To sugar, : . , ...		8	97	62

1807.	*Dr. Rufus Allen,* (Winthrop.)				
April 21.	To sundries;		3	6	05
June 6.	To sundries, ... ,		9	15	60

1807.	*Contra*	*Cr.*			Dolls.	Cts.
May 16.	By corn, ...		6		7	50

1807.	*Contra*	*Cr.*				
June 4.	By boards, ...		8		115	20

1807.	*Contra*	*Cr.*				
May 6.	By cash, ...		5		5	00

1807.	Dr.	Samuel King,		Dolls.	C
April 21.	To sundries;	4	2	7
June 6.	To salt & rum, ...	9	4	C
				6	7

1807.	Dr.	Benjamin Foster,			
April 24.	To indigo, ...	4	3	5
June 6.	To sundries, ...	9	5	8

1807.	Dr.	Charles Prince,			
June 3.	To balance from folio 3, . .		58	
6.	To brandy, ...	9	12	

1807.	*Contra*	*Cr.*			
				Dolls.	Cts.
June 6.	By transfer to R. Allen, . .		9	6	79
				6	79

	Contra	*Cr.*

	Contra	*Cr.*
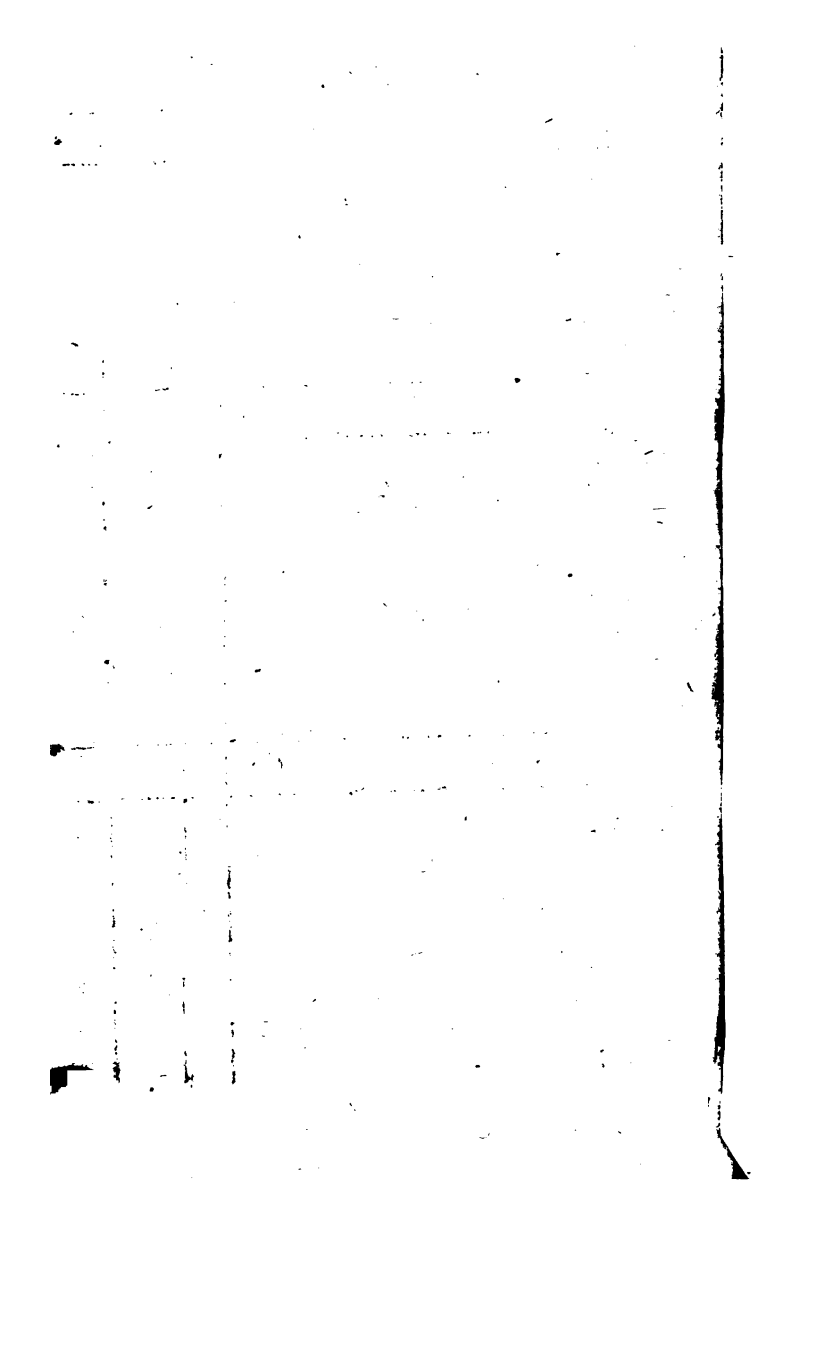
FORMS of NOTES, BILLS, RECEIPTS, &c.

. . . .

PROMISSORY NOTE.

Hallowell, June 6, 1807.

FOR value received, I promise to pay one hundred and twenty one dollars and fifty cents to George Rich or order, in sixty days, with interest.

HENRY WEST.

$121,50

Witness, GEO. SPELMAN.

PROMISSORY NOTE BY TWO PERSONS.

Hallowell, June 6, 1807,

For value received, we jointly and severally promise to pay fifty six dollars to A. B. or order, on demand with interest.

C. DAVIS.
E. FOX.

$56,00

Attest, C. HILL.

NOTE FOR BORROWED MONEY.

Borrowed and received of C. D. forty nine dollars, which I promise to pay on demand. ·

E. FOX.

$49,00

☞A promissory note having *order* inserted, may be endorsed from one person to another; and if *value received* is not mentioned, it is of no force.

P

INLAND BILL of EXCHANGE.

$1000,00   *Portland,* June 6, 1807.

Ten days after sight, pay to Mr. George Brown or order, one thousand dollars, for value received, and place it to my account without further advice (or as advised) from   Your humble servant,

 To *Mr. Geo. Rich,*   HENRY WEST.
   Boston.

FOREIGN BILL of EXCHANGE.

EXCHANGE for £.400 sterling.
        Hallowell, June 6, 1807.

Sixty days after sight (or at usance*) pay this my first bill of exchange, second and third of the same tenor and date not paid, to Mr. George Brown or his order, four hundred pounds sterling (exchange at four shillings and six pence per dollar) for value received, and place it (with or without) further advice, to the account of

        Your humble servant,

 Messrs. Neil & Thompson,   HENRY WEST.
  Merchants, Liverpool.

RECEIPT for MONEY paid on NOTE.

Hallowell, June 6, 1807. Received from William Grant (by the hands of Thomas Amory) sixty one dollars and fifty cents, which is endorsed on his note of May 16th, 1806.   SAMUEL PRINCE.

 $61,50

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* Usance is a customary time for the payment of foreign bills of exchange, circulating from one nation to another; and varies from 30 to 90 days, according to the custom of different countries.

RECEIPT for MONEY received on ACCOUNT.

June 6, 1707. Received from D. E. (by the hands of G. H.) forty dollars on account.   L. M.

$40,00

GENERAL RECEIPT.

June 6, 1807. Received of N. O. ten dollars and twenty nine cents, in full of all demands.

$10,29   N. B.

N. B. A general receipt will discharge all debts, except such as are on specialty, that is, bonds, bills and other instruments that may properly be called acts or deeds, viz. those that require to be executed in a solemn manner, where the sealing and delivery are the most essential parts of the act, and on that account can only be destroyed or cancelled by something of equal force, viz. some other specialty, such as a general release, &c. Neither will it discharge endorseable promissory notes, or inland bills.

. . . . . .

BANK DISCOUNT.

When a note is offered at a bank for discount, two endorsers are generally required, to the first of whom it is made payable : Thus A, having occasion to borrow money procures B and C as endorsers to his note, and offers it for discount in the following form.

$500,00   *Hallowell*, June 6, 1807.

For value received I promise to pay five hundred dollars to B or order, at the Hallowell and Augusta Bank, in fifty seven days, with customary grace.   •   A.

The method used among bankers in discounting notes, &c. is to find the interest of the sum from the date of the note to the time when it becomes due, including the days of grace ; the interest thus found is reckoned the discount, and is taken from the amount of the note at the time, before the person receives his money.

Grace denotes a term of three days, which custom has allowed to the borrower ; that is, though the note becomes due in fifty seven days, he may withold payment until the

sixtieth, for which reason the interest is reckoned for sixty days, notwithstanding the note should be paid the fifty seventh day.

. . . . . .

INVOICE of GOODS.

Boston, June 6, 1807.

Mr. N. Brown bought of

GEORGE RICH.

32 ells mode, ...	at 3s. 4d.	$17 78
64 yds. striped nankins, ...	1s. 6d.	16 00
28 - calicoe, ...	1s. 9d.	8 17
4 pieces muslin, ...	30s. . . .	20 00
56 yds. cotton cassimere, ...	2s. . .	18 67
20 pieces India cottons, ...	18s. . .	60 00
25 - plain nankins, ...	6s. 6d.	27 08
2 doz. cotton hose (men's) . . .	66s. . .	22 00

$189 70

Rec'd payment by his note at 60 days,

GEO. RICH.

ACCOUNT RENDERED.

Mr. RICHARD LEWIS

1807.   To A—— B——— Dr.

April 8. To 2 yds. superfine cloth, . . . at $6,00		$12 00
- 4 - blk. cassimere, . . .	2,50	10 00
- 1½ doz. buttons, ...	,92	1 38
- 1 - small do.		37
- 4 skeins silk, ...	,06¼	25
- 4 sticks twist, ...	,06¼	25
24. - 5 pieces In. cottons, 22 yds. ea. 4,33⅓		21 67
- 7 yds. cotton cambric, . . .	,75	5 25
- 6 - col'd do.	,50	3 00
May 18. -14 - do. do.	,50	7 00
- 7 - linen, ...	,92	6 44
- 1 oz. thread, No. 40		41
- 2 pair morocco shoes, . . .	1,08	2 16
- 1 oz. indigo, ...		25

Hallowell, June 6, 1807. . . $70 43

Rec'd payment for A—— B——

GEORGE NORTH.

Dr.   Mr. *Nehemiah Gordon* in account current with *Hiram Weston*,   Cr.

1807.		$.	C.	1807.		$.	C.
April 16.	To 2 pieces India cottons, 22 yds. each, at 26s.	8	67	April 18.	By cash received of J. Thomas,	5	00
	1¾ yd. Marseilles vesting, at 8s.	2	00	June 4. - cash to balance,		7	96
June 3. - 1 lb. bohea tea,		0	46				
- 4 lb. ditto, ... at 2s. 9d.		1	83				
		12	96			12	96

Hallowell, June 4, 1807.
Errors excepted,
HIRAM WESTON.

P 2

A Common INDENTURE *to bind an Apprentice.*

THIS indenture witnesseth, That A. B. of, &c. hath put and placed, and by these presents doth put and bind out his son C. D. and the said C. D. doth hereby put, place and bind out himself, as an apprentice to R. P. to learn the art, trade or mystery of   The said C. D. after the manner of an apprentice to dwell with and serve the said R. P.   from the day of the date hereof, until the   day of   which will be in the year of our Lord one thousand eight hundred and   at which time the said apprentice, if he should be living, will be twenty one years of age : During all which time or term, the said apprentice his said master well and faithfully shall serve ; his secrets keep, and his lawful commands every where at all times readily obey ; he shall do no damage to his said master, nor wilfully suffer any to be done by others ; and if any to his knowledge be intended, he shall give his master seasonable notice thereof. He shall not waste the goods of his said master, nor lend them unlawfully to any : At cards, dice or any other unlawful game he shall not play ; fornication he shall not commit nor matrimony contract, during the said term ; taverns, alehouses, or places of gaming, he shall not haunt or frequent : From the service of his said master he shall not absent himself ; but in all things and at all times he shall carry and behave himself as a good and faithful apprentice ought, during the whole time or term aforesaid.

And the said R. P. on his part, doth hereby promise, covenant and agree to teach and instruct the said apprentice, or cause him to be taught and instructed, in the art trade or calling of a   by the best way or means he can, and also to teach and instruct the said apprentice, or cause him to be taught and instructed to read, write and cypher as far as the rule of three, if the said apprentice be capable to learn ; and shall well and faithfully find and provide for the said apprentice good and sufficient meat, drink, clothing, lodging, and other necessaries fit and convenient for such an apprentice during the term aforesaid, and at the expiration thereof shall give unto the said apprentice two suits of wearing apparel,

one suitable for the LORD's days, and the other for work-ing days.

· In testimony whereof, the said parties have hereunto in-terchangeably set their hands and seals, the   day of   in the year of our LORD one thousand eight hun-dred and

  Signed, sealed and delivered   (Seal)
      'in presence of us   (Seal)
                                              (Seal)

WARRANTEE DEED.

KNOW all men by these presents, That I, A. B. of, &c. in consideration of the sum of   paid me by C. D. of, &c. the receipt whereof I do hereby acknowledge, do hereby give, grant, bargain, sell and convey unto the said C: D. his heirs and assigns forever. *(Here insert the premises.)* *To have and to hold*, the said granted and bargained premises, with the privileges and appurtenances thereof, to him the said C. D. his heirs and assigns forever ; to his and their use and behoof forever. And I the said A. B. for myself, my heirs, executors and administrators, do covenant with the said C. D. his heirs and assigns, that I am lawfully seized in fee of the premises ; that they are free of all incumbrances ; that I have good right to sell and convey the same to the said C. D. to hold as aforesaid ; and that I will warrant and defend the same to the said C.D. his heirs and assigns forever, against the lawful claims and demands of all persons.

In witness whereof I have hereunto set my hand and seal the   day of, &c.

                                        A. B.   *(Seal)*

! *Signed, sealed and delivered,*
     in presence of us, &c.

QUITCLAIM DEED.

KNOW all men by these presents, That I, A. B. of, &c. in consideration of the sum of   to me paid by C. D. of, &c. the receipt whereof I do hereby acknowl-

edge, have remissed, released and forever quitclaimed, and do by these presents remiss, release and forever quitclaim unto the said C. D. his heirs and assigns forever. *(Here insert the premises.)* *To have and to hold* the same, together with all the privileges and appurtenances thereunto belonging, to him the said C. D. his heirs and assigns forever.

In witness, &c.

MORTGAGE DEED.

KNOW all men by these presents, that I, A. B. of, &c. in consideration of the sum of   paid to me by C. D. of, &c. the receipt whereof I do hereby acknowledge, do hereby give, grant, bargain, sell and convey unto the said C. D. his heirs and assigns forever. *(Here insert the premises.)* *To have and to hold* the said granted and bargained premises with the privileges and appurtences thereof, to the said C. D. his heirs and assigns, to his and their use and behoof forever. And I the said A. B. for myself, my heirs, executors and administrators, do covenant with the said C. D. his heirs and assigns, that I am lawfully seized in fee of the premises, that they are free of all incumbrances, that I have good right to sell and convey the same to the said C. D. to hold as aforesaid, and that I will warrant and defend the same to the said C. D. his heirs and assigns forever, against the lawful claims and demands of all persons.

Provided nevertheless, That if I the said A. B. my heirs, executors, or administrators, shall well and truly pay to the said C. D. his heirs, executors, administrators or assigns, the full and just sum of   on or before the day of   next, *(or* which will be in the year of our Lord   *)* with lawful interest for the same until paid, then this deed, [as also a certain bond, *(or note, as the case may be)* bearing even date with these presents, given by me to the said C. D. conditioned to pay the same sum and interest at the time aforesaid] shall be void ; otherwise shall remain in full force and virtue.

In witness whereof, &c.

A TABLE *for reducing Shillings and Pence into Cents and Mills.*

	Shil. 0	Shil. 1	Shil. 2	Shil. 3	Shil. 4	Shil. 5
Pen.	cts. m.	cts. m.	cts. m.	cts. m.	cts. m.	cts. m.
0		16 7	33 3	50 0	66 7	83 3
1	1 4	18 1	34 7	51 4	68 1	84 7
2	2 8	19 5	36 1	52 8	69 5	86 1
3	4 2	20 9	37 5	54 2	70 9	87 5
4	5 6	22 3	38 9	55 6	72 3	88 9
5	7 0	23 7	40 3	57 0	73 7	90 3
6	8 3	25 0	41 6	58 3	75 0	91 6
7	9 7	26 4	43 0	59 7	76 4	93 0
8	11 1	27 8	44 4	61 1	77 8	94 4
9	12 5	29 2	45 8	62 5	79 2	95 8
10	13 9	30 6	47 2	63 9	80 6	97 2
11	15 3	32 0	48 6	65 3	82 0	98 6

EXAMPLE.—Reduce 3s. 6d. to cents and mills. Look for 3s. at the head of the column, and 6 under pence at the left hand side; then casting your eye along in that line until you come to the 3s. column, you have 58 cents 3 mills, the answer.

———◆———

A TABLE *in which the gold coins of Great Britain and Portugal are reduced to an equivalent value in dollars and cents.*

gr	ct	gr	ct	pwt	dl ct	pwt	dol. ct.
1	3	13	48	1	0,89	11	9 78
2	7	14	51	2	1,78	12	10 67
3	11	15	55	3	2,67	13	11 55
4	14	16	59	4	3,55	14	12 44
5	18	17	63	5	4,44	15	13 33
6	22	18	67	6	5,33	16	14 22
7	25	19	70	7	6,22	17	15 11
8	29	20	74	8	7,11	18	16 00
9	33	21	78	9	8,00	19	16 89
10	37	22	81	10	8,89	20	17 78
11	40	23	85				
12	44						

A TABLE *in which the gold coins of France and Spain are reduced to an equivalent value in dollars and cents.*

gr	ct	gr	ct	pwt	dl ct	pwt	dl ct
1	3	13	47	1	0,87	11	9,63
2	7	14	51	2	1,75	12	10,51
3	11	15	55	3	2,63	13	11,39
4	14	16	58	4	3,50	14	12,26
5	18	17	62	5	4,38	15	13,14
6	22	18	66	6	5,25	16	14,01
7	25	19	69	7	6,13	17	14,89
8	29	20	73	8	7,01	18	15,76
9	33	21	76	9	7,88	19	16,64
10	36	22	80	10	8,76	20	17,52
11	40	23	84				
12	44						

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